High-Fidelity, Time-Efficient Modeling of Articulated, Flexible Multibody Spacecraft

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Examples of Flexible Multibody Spacecraft in Large Overall Motion: Configuration Change, Spin, Slewing, and Antenna "Extrusion"

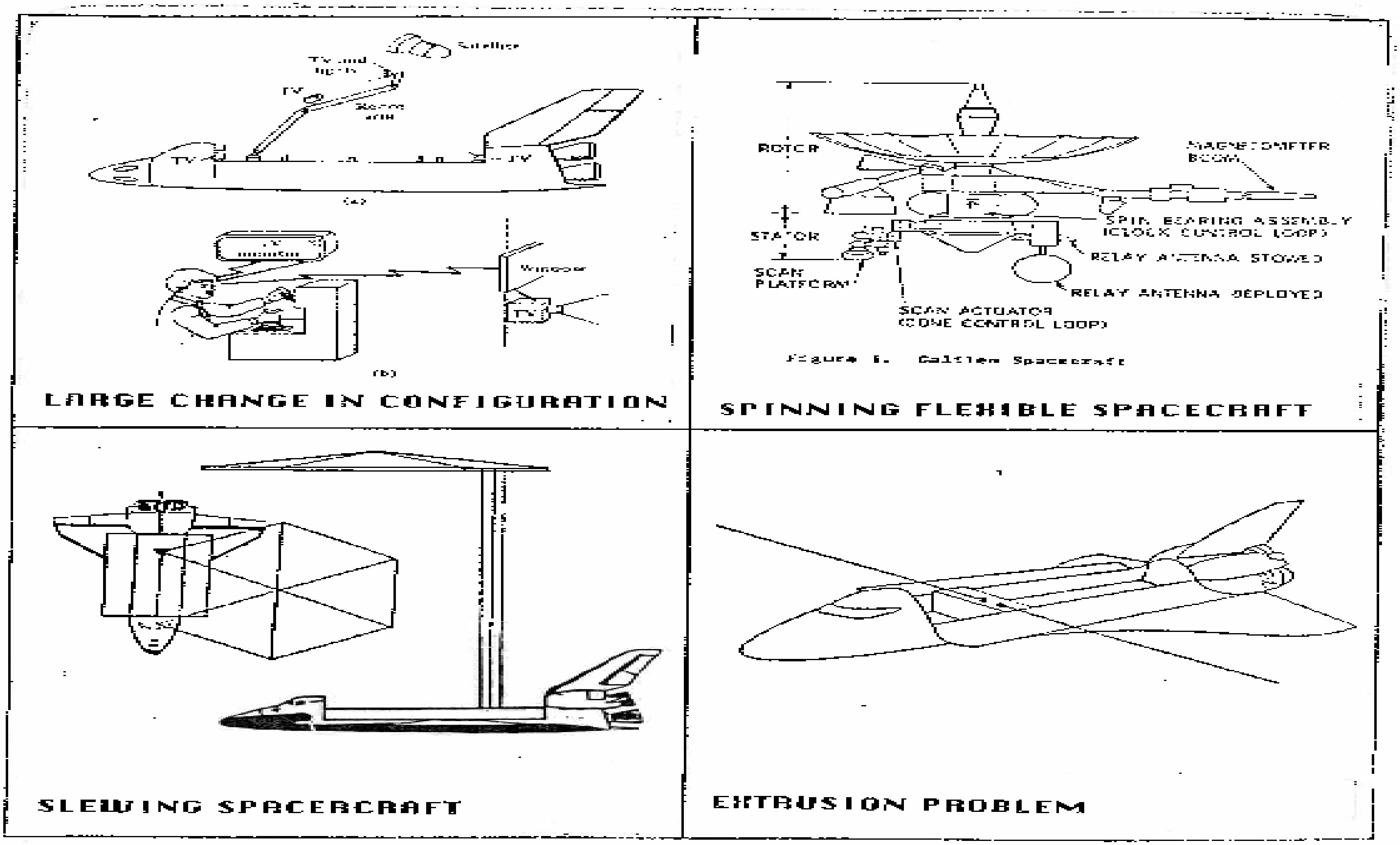


Fig.10 Representative Systems Undergoing Stationkeeping, Constant Spin, Slewing, and Spin-up Motions

JPL's Galileo Spacecraft : A Flexible Multibody System

1. HG flexible antenna rotates w.r.t. flexible stator
2. RTG booms swivel w.r.t. HG antenna about rotary hinges to provide wobble damping
3. Scan platform rotates w.r.t. stator in 2 dof hinges

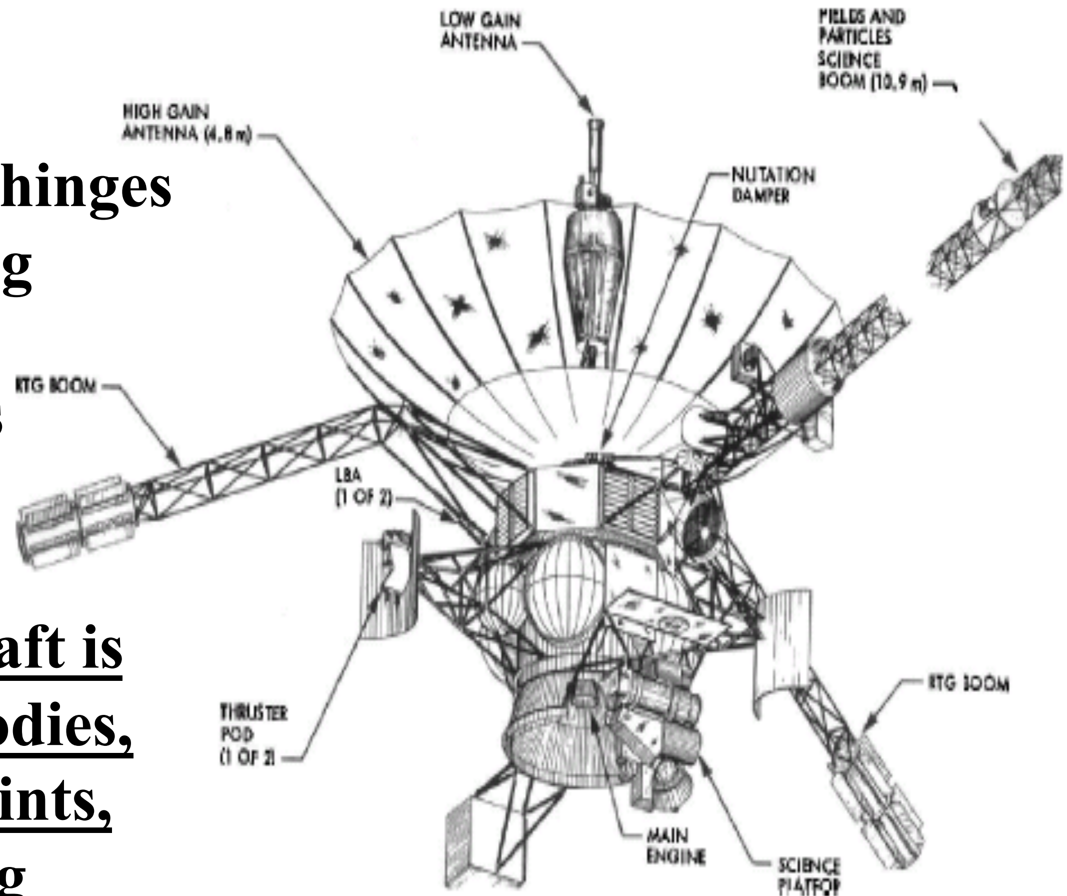


Fig. 1 Galileo spacecraft built by the Jet Propulsion Laboratory.

Studied Jupiter's Atmoshere

Essence: <u>a typical spacecraft is made of several flexible bodies, connected by rotational joints, with the bodies undergoing large rotation with small elastic vibration</u>

Kane's Method of Deriving the <u>Simplest Eqns of Motion by the Least Labor</u>: Example--50 DOF System of 4 Flex Bodies with Rotary Joints

Kane, T. R. and Levinson, D. A., "Formulation of Equations of Motion for Complex Spacecraft," Jour. of Guidance and Control, Mar-Apr 1980, pp. 99-112.

<u>Simplifying</u> motion variables $u_j = \sum_{k=1}^{n} A_{jk}\,\dot{q}_k + D_j, \quad \Rightarrow \dot{q}_k = \sum_{k=1}^{n} W_{kj}\,u_j + X_k \quad j = 1,\ldots,n$

$$\mathbf{v}^k = \sum_{j=1}^{n} \frac{\partial \mathbf{P}^k}{\partial q_j}\dot{q}_j + \frac{\partial \mathbf{P}^k}{\partial t} = \sum_{j=1}^{n} \frac{\partial \mathbf{v}^k}{\partial u_j} u_j + \mathbf{B}^k$$

Velocity is in terms of *j*-th <u>Partial Velocities</u> , $\frac{\partial v^k}{\partial u_j}$ which are <u>functions of gen. coords, q's</u>

$$\mathbf{a}^k = \sum_{j=1}^{n} \frac{\partial \mathbf{v}^k}{\partial u_j}\dot{u}_j + \mathbf{C}^k$$

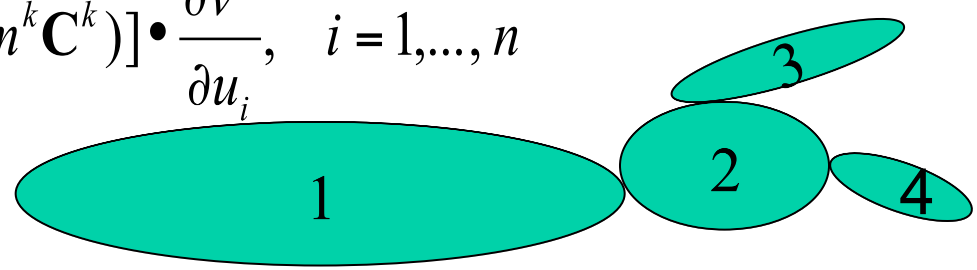
Acceleration of generic point *k*

$$0 = \sum_{k=1}^{NP} [\mathbf{F}^k + (-m^k \mathbf{a}^k)] \bullet \frac{\partial \mathbf{v}^k}{\partial u_i} \quad , i = 1,\ldots,n$$

Use of <u>dot-products</u> make <u>Kane's Equations of least labor</u>

$$\sum_{k=1}^{NP} m^k \sum_{j=1}^{n} [\frac{\partial v^k}{\partial u_i} \bullet \frac{\partial v^k}{\partial u_j}]\dot{u}_j = \sum_{k=1}^{NP} [\mathbf{F}^k + (-m^k \mathbf{C}^k)] \bullet \frac{\partial v^k}{\partial u_i}, \quad i = 1,\ldots,n$$

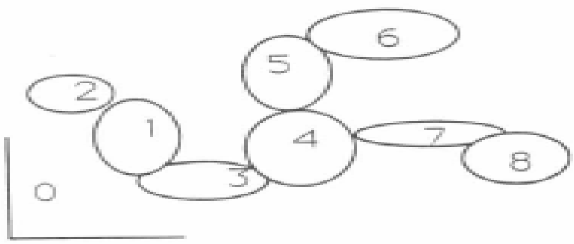
$$\left[M(q)\right]\left\{\dot{U}\right\} = \left\{R\right\}$$

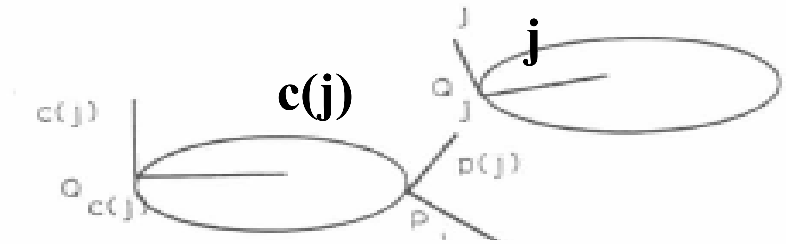


For 50 dof system, form, decompose & solve 50x50 dense, time-varying matrix 4 times per RK step; Unmodified this approach is expensive !

3-Step Recursion of Kane's Eqs for Block-diagonal Matrix Eqs*

Step 1) Form Kinemtics Eqns going <u>Forward</u> <u>from Body 1 to Body n</u>





ang. vel. j-frame

$$\omega^j = C^T_{c(j),j}\left[\omega^{c(j)} + \varphi^{c(j)}(P_j)\dot{\eta}^{c(j)} + C_{c(j),p(j)}G^j\dot{\theta}^j\right]$$

inboard body ang vel, modal slope rates, gimbal rotation rates

velocity of Qj

$$\nu^{Q_j} = C^T_{c(j),j}\Big\{\nu^{Q_{c(j)}} + \tilde{\omega}^{c(j)}\left[r^{Q_{c(j)}P_j} + \phi^{c(j)}(P_j)\eta^{c(j)}\right] + \phi^{c(j)}(P_j)\dot{\eta}^{c(j)} + C_{c(j),p(j)}L^j\dot{\tau}^j + \Big[\tilde{\omega}^{c(j)} + \widetilde{\varphi^{c(j)}(P_j)\dot{\eta}^{c(j)}}\Big]C_{c(j),p(j)}L^j\tau^j\Big\}$$

inboard body hinge Pj-velocity, plus account for sliding joint

***Banerjee, A. K., "Block-Diagonal Equations for Flexible Multibody Dynamics with Geometric Stiffness and Constraints", JGCD, Nov. - Dec., 1993, pp. 1092-1100.**

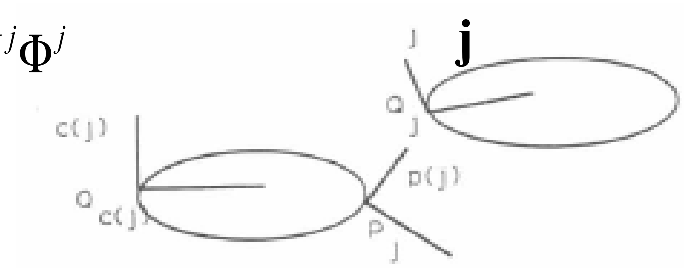
Block-Diagonal Eqns (continued)

Step 2) Form Dynamical Eqns for <u>Body n</u> <u>Going Backward to body1</u>

<u>Write Eqs. for Body j</u> <u>Vibration for Base Accl: invert nmodes x nmodes</u> matrix

$$E^j \ddot{\eta}^j = A^j \begin{Bmatrix} a_0^{Q_j} \\ \alpha_0^j \end{Bmatrix} + Y_1^j$$

$$E^j = \Phi^{j^t} M^j \Phi^j$$



$$\begin{Bmatrix} a_0^{Q_j} \\ \alpha_0^j \end{Bmatrix} = \begin{Bmatrix} \hat{a}_0^{Q_j} \\ \hat{\alpha}_0^j \end{Bmatrix} + R^j \begin{Bmatrix} \ddot{\tau}^j \\ \ddot{\theta}^j \end{Bmatrix}$$

<u>Write Rotn / Tran</u> <u>Newton-Euler Eqs for body j hinge:</u> <u>invert matrix of size 1 to 6</u>

$$\begin{Bmatrix} \ddot{\tau}^j \\ \ddot{\theta}^j \end{Bmatrix} = -{\nu^j}^{-1} R^{jT} \left[M_3^j \begin{Bmatrix} \hat{a}_0^{Q_j} \\ \hat{\alpha}_0^j \end{Bmatrix} + Y_2^j \right] + {\nu^j}^{-1} \begin{pmatrix} f_h^j \\ t_h^j \end{pmatrix}$$

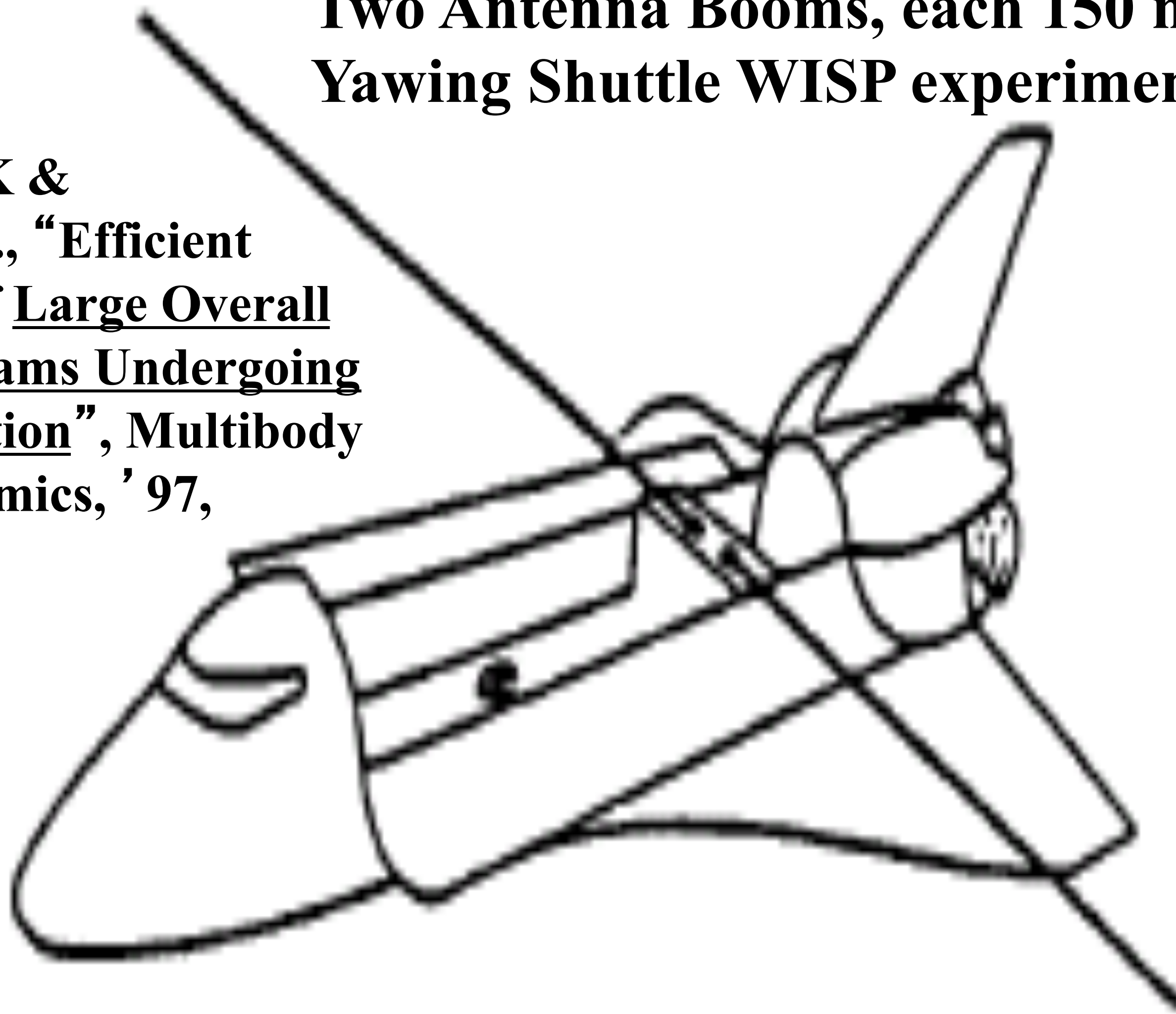
$$\nu^j = R^{j^t} m^j R^j$$

<u>Step 3)After eqns for body #1 are formed, go forward one more time to explicitly “uncover” eqns for bodies 2,…,n one at a time</u>

Applications: Degenerate Case of Systems of Rigid Bodies

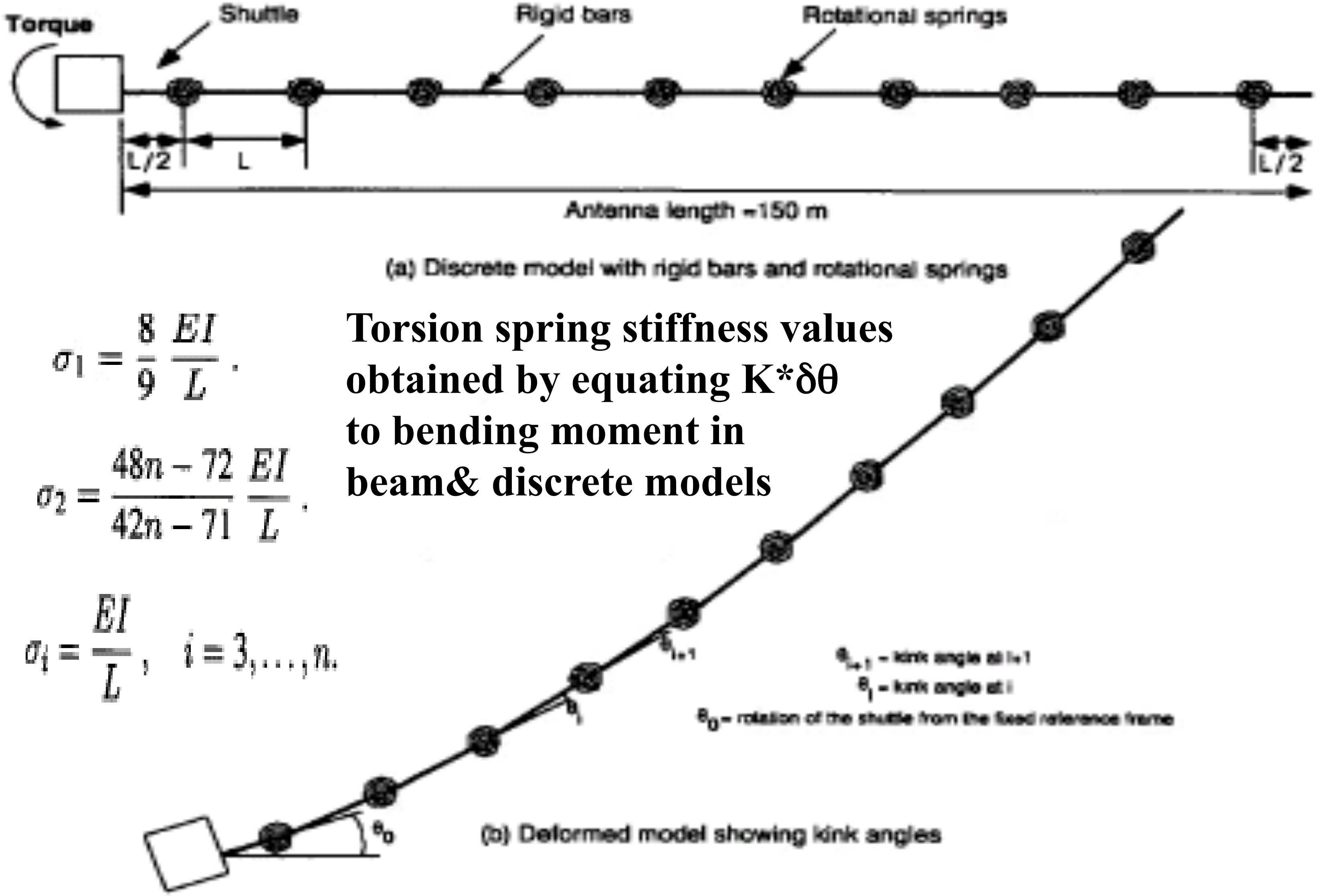
Two Antenna Booms, each 150 m long, for Yawing Shuttle WISP experiment

Banerjee, A.K & Nagarajan, S., "Efficient Simulation of <u>Large Overall Motion of Beams Undergoing Large Deflection</u>", Multibody System Dynamics, '97, pp.113-126.



Shuttle Waves in Space Plasma (WISP) Experiment

Elastic Beam Modeled as Many Rigid Rods Pin-Connected by Rotational Springs, for Large Bending Simulation of 150 m Antenna



$$\sigma_1 = \frac{8}{9}\frac{EI}{L}\,.$$

$$\sigma_2 = \frac{48n-72}{42n-71}\frac{EI}{L}\,.$$

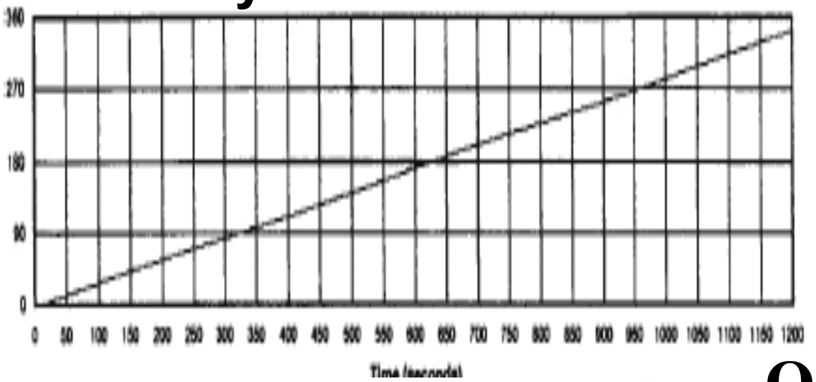
$$\sigma_i = \frac{EI}{L}\,, \quad i = 3,\ldots,n.$$

Torsion spring stiffness values obtained by equating K*δθ to bending moment in beam& discrete models

Rigid Body Order-n vs. Nonlinear FEM for Shuttle WISP Antenna: Large Yaw Angle, Root Bending Moment, Tip Deflection, CPU sec

Shuttle yaws ~1 rev in 20 min

Yaw (deg)



CPU sec

Table I.

Formulation	Number of elements	Step size (sec)	CPU time (sec)
order-n	10	1.0	24
order-n	10	0.5	40.9
finite element	10	1.0	890

O(n) Model 36 times faster than FEM

Bending Moment

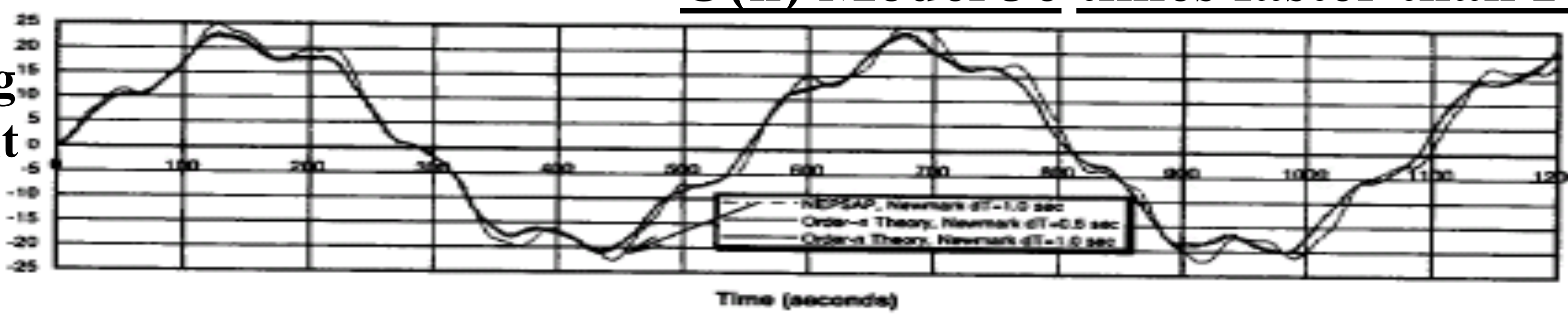
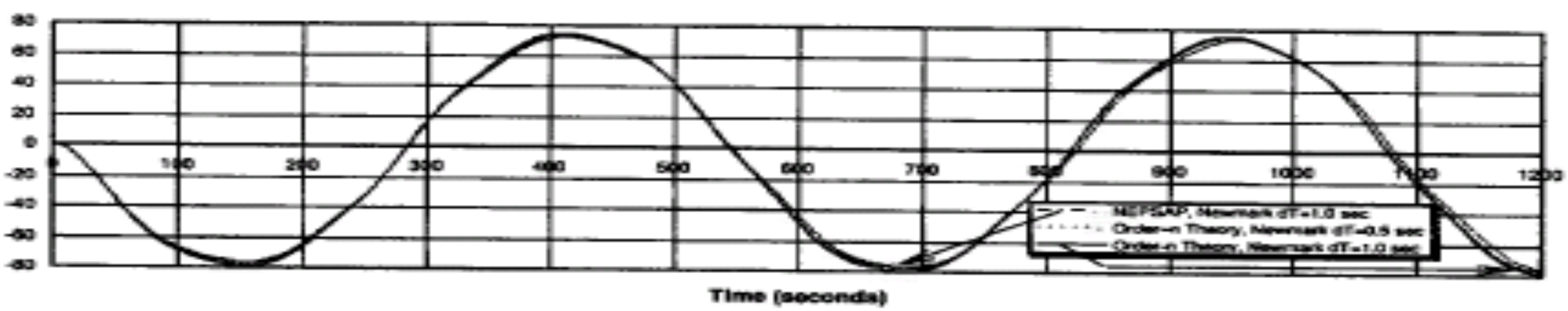


Figure 6. Beam root bending moment, 10 element solution.

Tip Defl 80 m for a 150 m beam

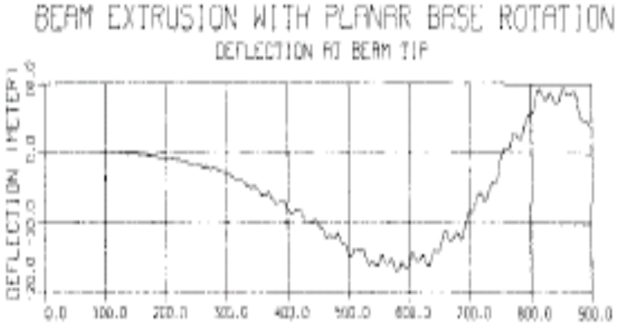


Deployment & Retraction Problems : <u>Variable-n O(n) Formulation</u> of two 150 m Booms from Yawing Shuttle [Banerjee, JGCD,1992]

Deployment (telescoping)

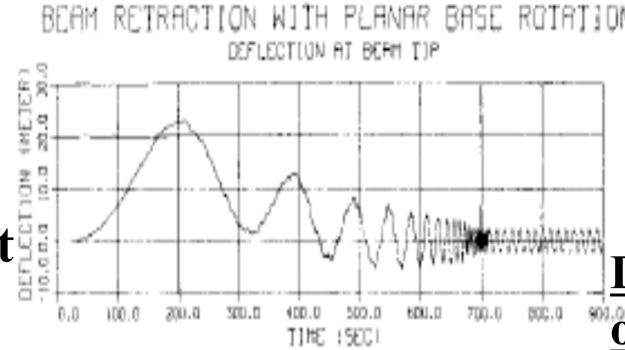
Retraction (spaghetti-eating)

Tip Deflec max 16 ft



a)

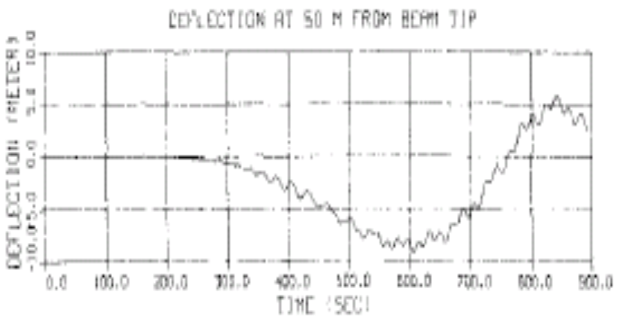
Tip Deflec max 23 ft



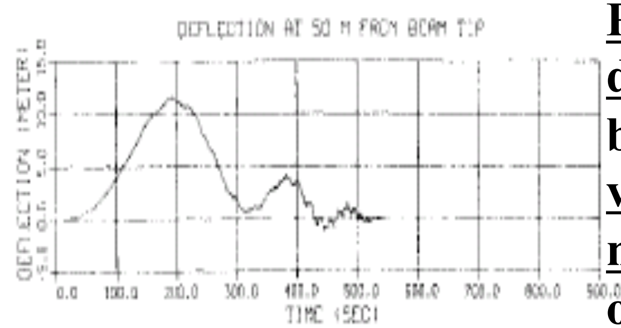
a)

<u>Deploying or Retieval described by varying number of rigid rods</u> in play

Deflec 50 m from tip

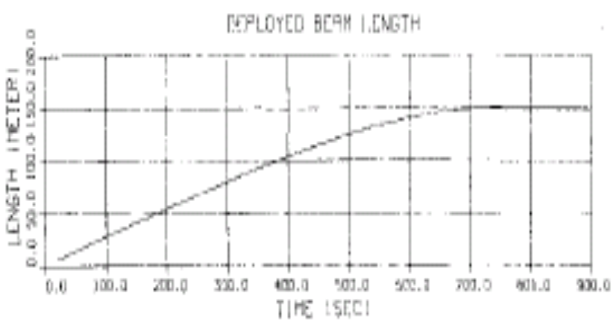


b)



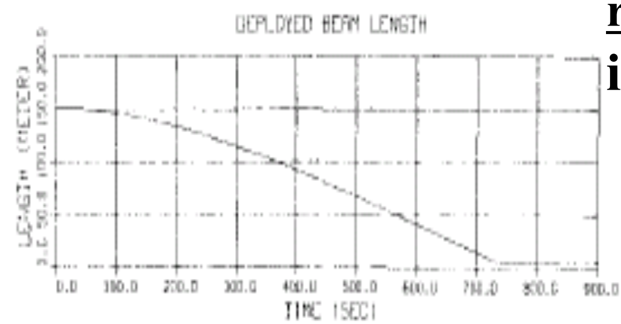
b)

Boom Length



c)

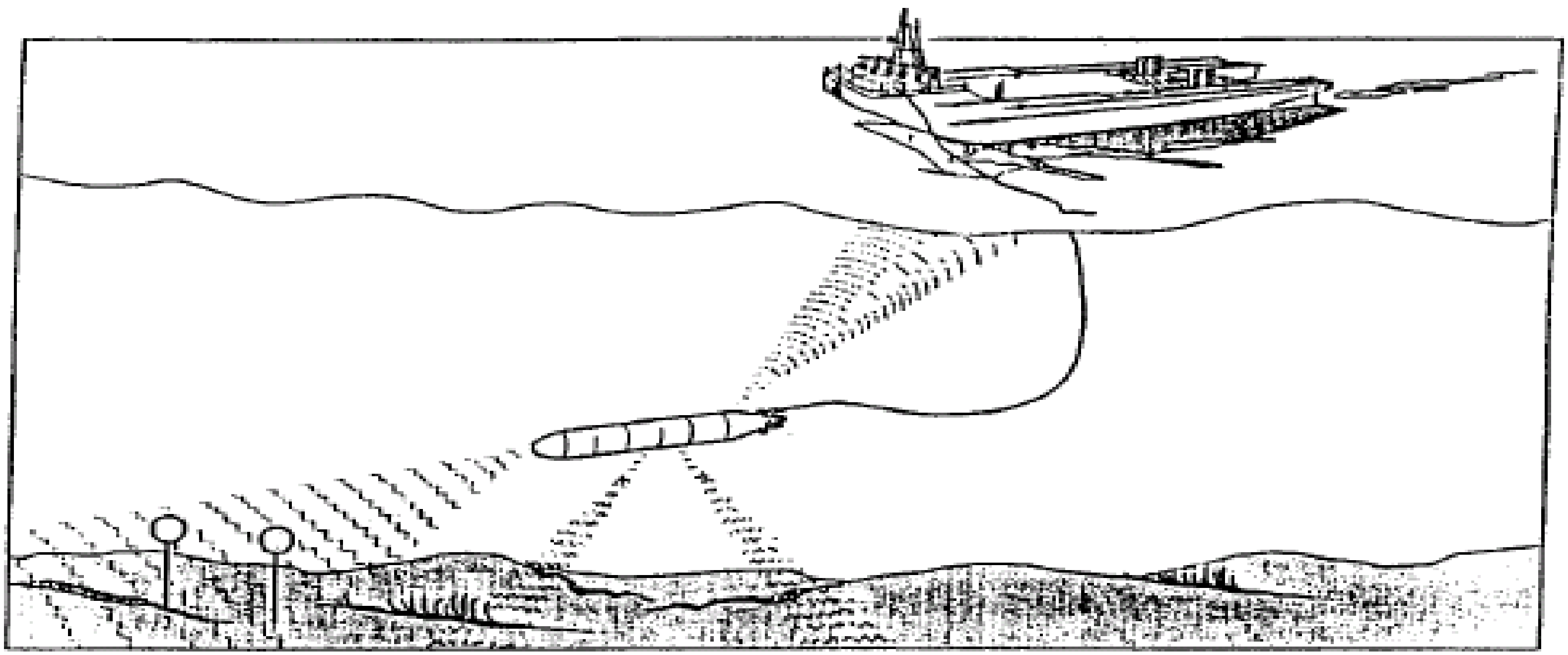
12 min to deploy/ retrieve 150 m boom



c)

<u>WISP Boom Bending due to Coriolis Force</u>

Modeling Cable Deployment Dynamics in Towing of Underwater Powered Device Connected by Cable to Ship



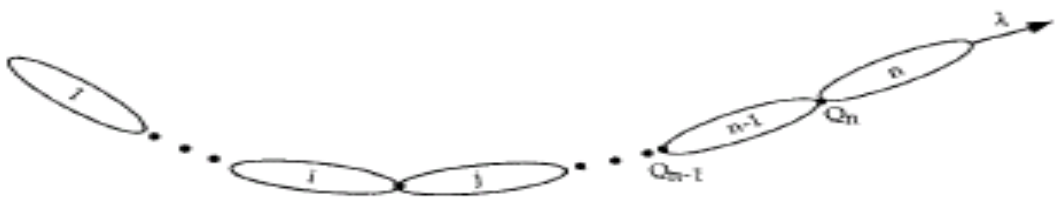


Fig. 2 Discretization of cable.

<u>Cable modeled by pinned links with no joint stiffness</u>: Banerjee &Do, JGCD,' 94

Cable Length & Tension Change in Deployment / Retrieval vs. Time

deployment **600 m in 600 sec** **retrieval**

length

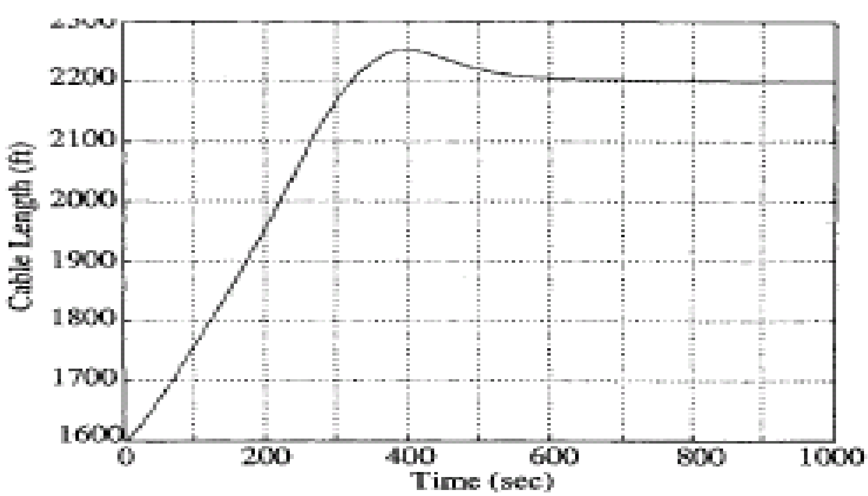


Fig. 7a **Cable length vs time during deployment.**

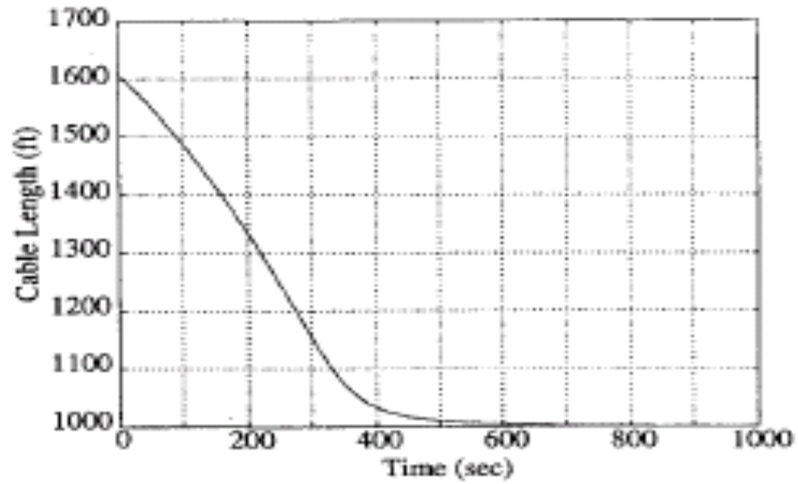


Fig. 9a **Cable length vs time during retrieval.**

tension

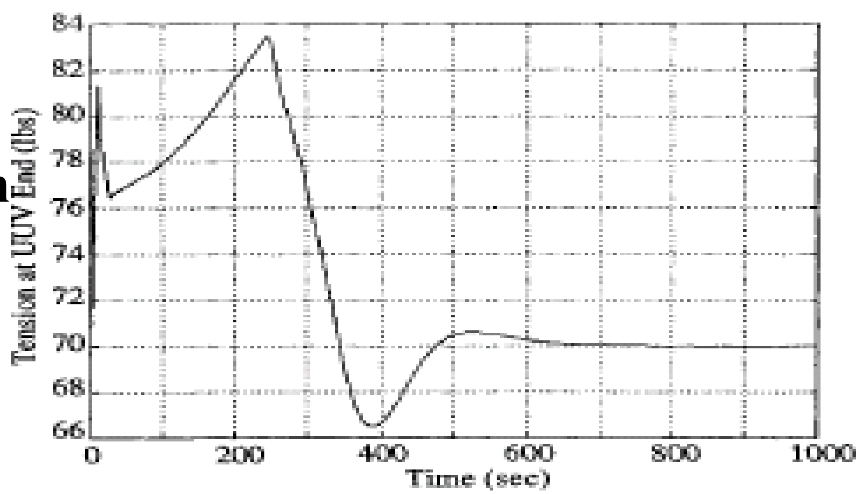


Fig. 7b **Cable tension vs time during deployment.**

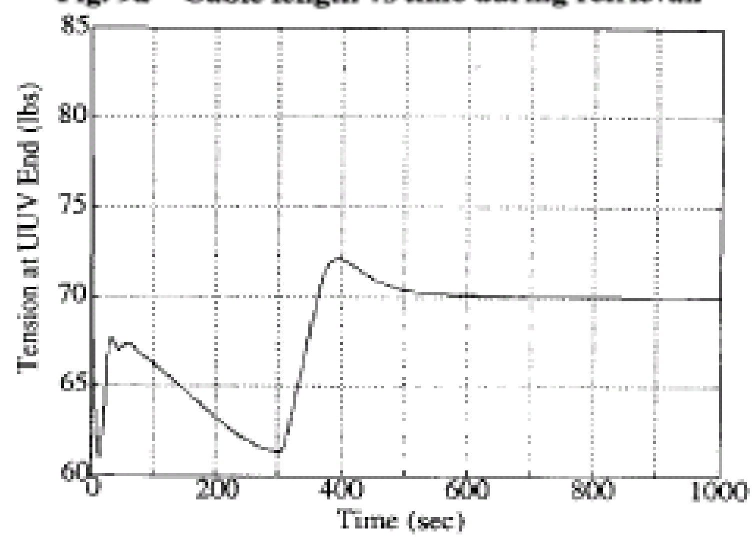
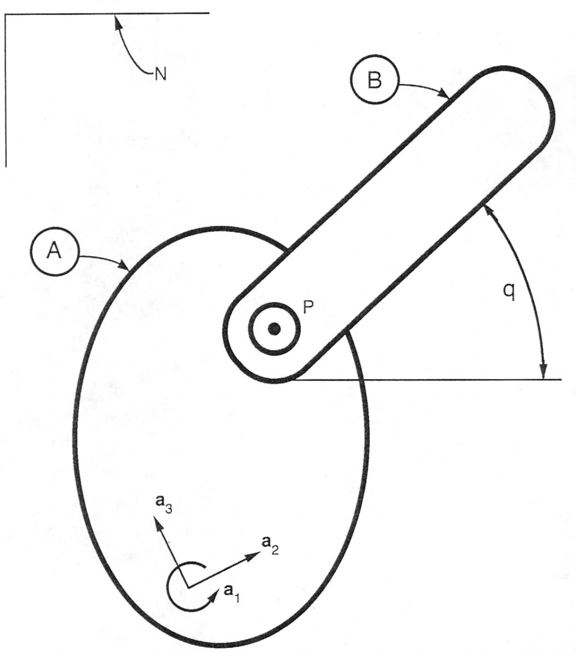


Fig. 9b **Cable tension vs time during retrieval.**

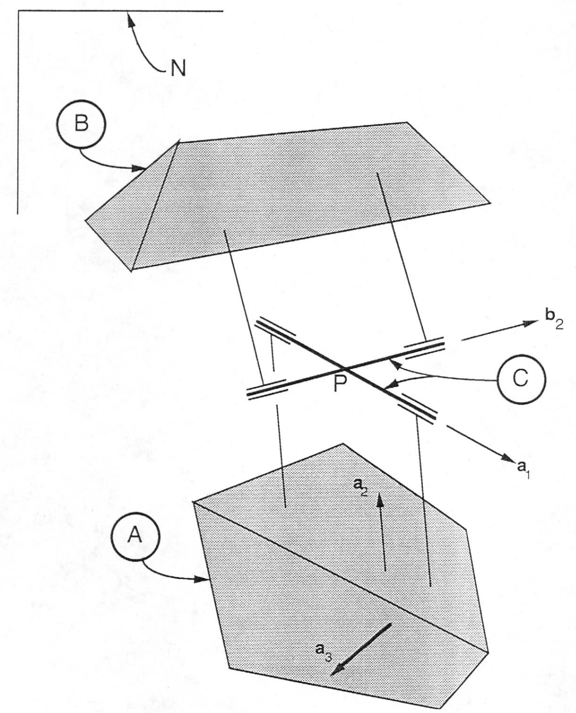
Monitoring internal force with O(n) formulation

Simplifying Rotational Generalized Speeds (Mitiguy & Kane)

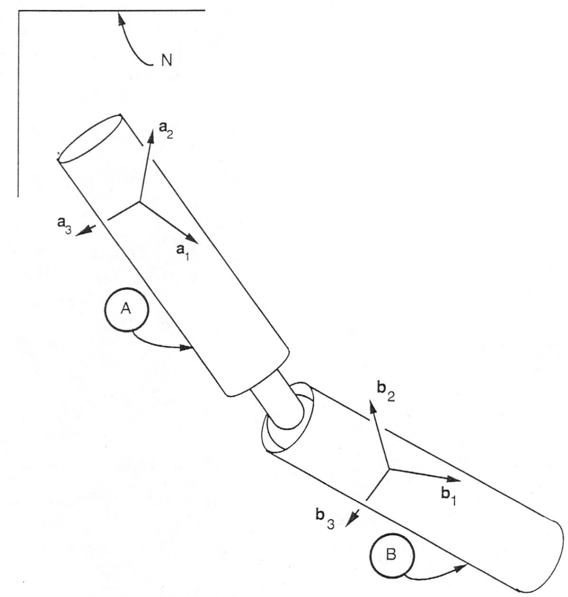
Revolute Joint



2 DOF Az-El Gimbal



3 DOF Spherical Joint



Generalized Speeds

$$u_1 = {}^{N}\boldsymbol{\omega}^{B} \bullet \mathbf{a}_1$$

$$u_1 = {}^{N}\boldsymbol{\omega}^{C} \bullet \mathbf{a}_1$$

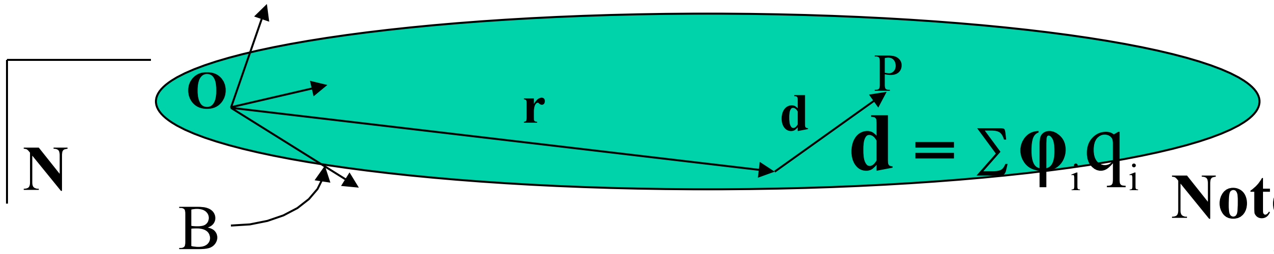
$$u_2 = {}^{N}\boldsymbol{\omega}^{B} \bullet \mathbf{b}_2$$

$$\mathrm{u}_\mathrm{i} = {}^{\mathrm{N}}\boldsymbol{\omega}^{\mathrm{B}} \cdot \mathbf{b}_\mathrm{i}$$

$$(\mathrm{i} = 1,2,3)$$

Hinge relative rotation angle rates are <u>not</u> the best choices

Efficient Generalized Speeds (D' Eleuterio & Hughes) for Vibration of Elastic Bodies in Large Rotation



$$\mathbf{d} = \sum \boldsymbol{\varphi}_i q_i$$

Note: use of vibration modes, universally done, is an act of premature linearization

$$^N\mathbf{v}^P = {}^N\mathbf{v}^O + {}^N\boldsymbol{\omega}^B \times \left(\mathbf{r} + \sum \boldsymbol{\varphi}_i q_i\right) + \sum \boldsymbol{\varphi}_i \dot{q}_i$$

Propose Gen. Speeds $\sigma_i \quad \Rightarrow \qquad \sum_i \phi_i \sigma_i = {}^N\omega^B \times \sum_i \phi_i q_i + \sum_i \phi_i \dot{q}_i$

$$^N\mathbf{v}^P = {}^N\mathbf{v}^O + {}^N\boldsymbol{\omega}^B \times \mathbf{r} + \sum \boldsymbol{\varphi}_i \sigma_i$$

Velocity expression free of modal coordinates !

kinematics eqs: $$\sum_i \int \phi_i \phi_j dm(\sigma_j - \dot{q}_j) = {}^N\omega^B \times \sum_i \int \phi_j \phi_i dm q_j$$

Consequences of q-independence of Velocity Expression on Mass Matrix in Kane's Equations for a Free-Flying Flexible Body

$$^{N}\mathbf{v}^{P} = \sum_{i}^{3} u_i \mathbf{b}_i + \sum_{i}^{3} u_{3+i} \mathbf{b}_i \times \mathbf{r} + \sum_{\mathbf{i}}^{n} \boldsymbol{\phi}_{\mathbf{i}} u_{6+i} \Rightarrow \frac{\partial\, ^{N}\mathbf{v}^{P}}{\partial u_i} \neq f(q) \quad ,(i = 1,\ldots,6+n)$$

Partial velocities are now free of generalized coordinates

Kane's Equations, obtained by integrating over body, are:

$$\int \sum_{j=1}^{n} \frac{\partial \mathbf{v}}{\partial u_i} \bullet \frac{\partial \mathbf{v}}{\partial u_j}\, dm \;\; \dot{u}_j = -\int C \bullet \frac{\partial \mathbf{v}}{\partial u_i}\, dm - \sum \left[\frac{\partial P}{\partial q_j} + \frac{\partial D}{\partial \dot{q}_j} \right] \frac{\partial \dot{q}_j}{\partial u_i} + \sum_{k=1}^{NF} F_k^{ext} \bullet \frac{\partial \mathbf{v}^k}{\partial u_i} \quad (i = 1,\ldots,6+n)$$

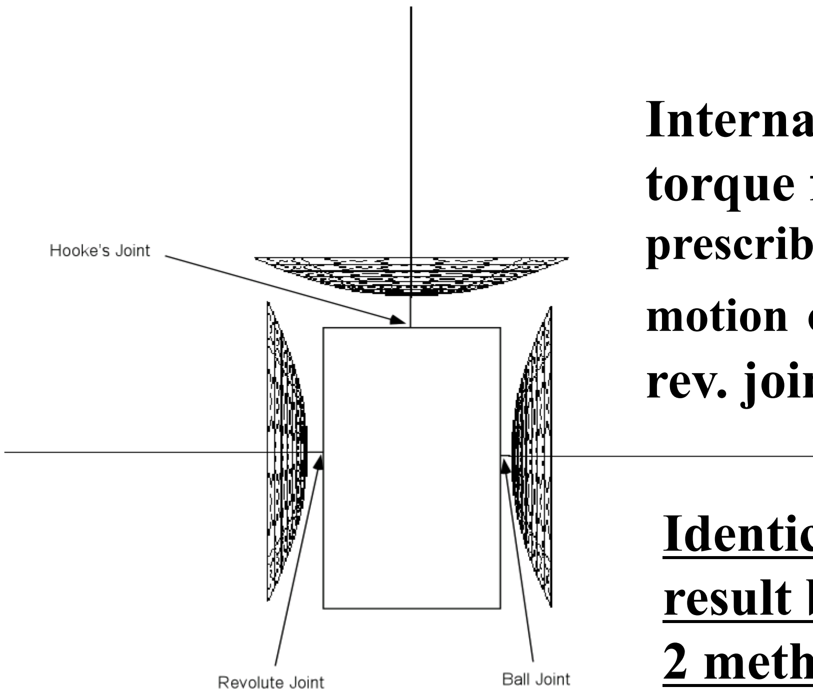
$$M \dot{U} = R$$

<u>Mass matrix in LHS is now constant</u>

<u>Facilitates fast computation for single or terminal flexible body.</u>

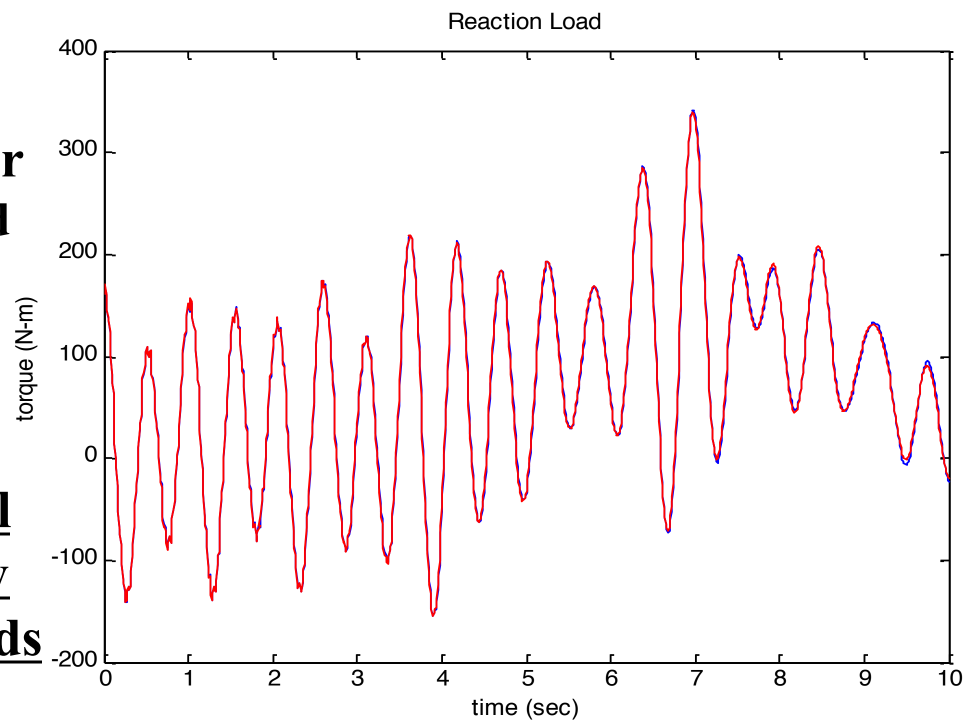
Application: System of 4 Hinge-Connected Flexible Bodies, Kane's vs. Recursive Method with Efficient Gen. Speeds--- Loads Analysis

Constraint torque at a joint in a 4-flex-body system with 1-, 2-, 3-rotation joints: [<u>Int. Cong. Theo. & App. Mech, Warsaw, 04</u>]



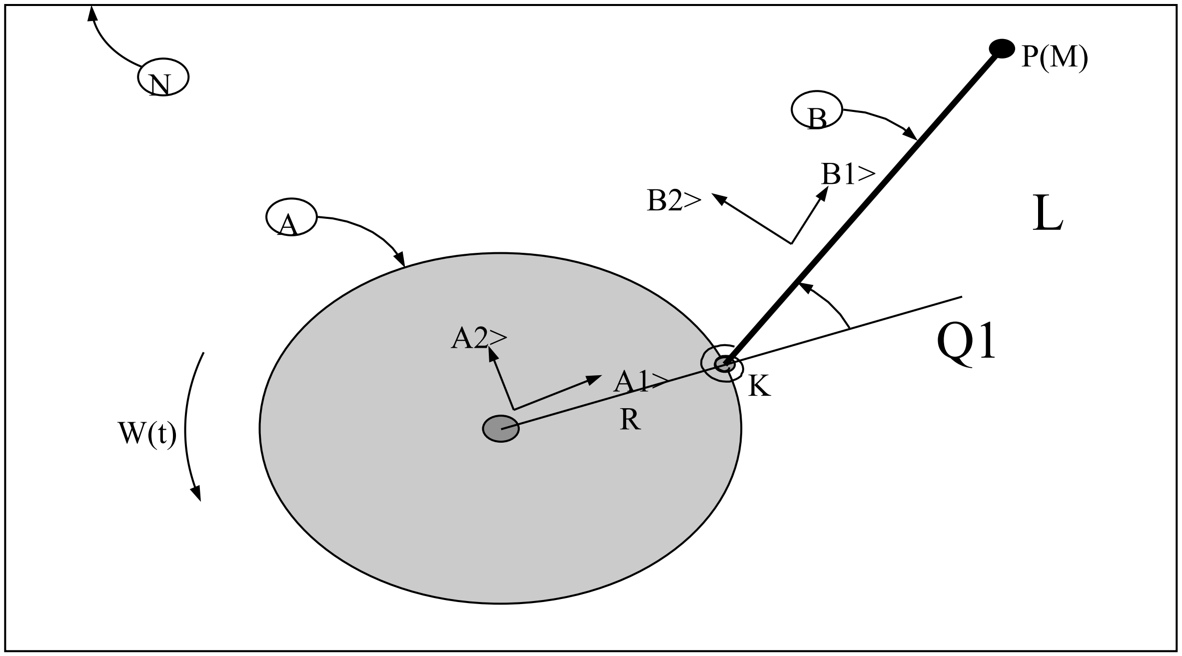
Internal torque for prescribed motion of rev. joint

<u>Identical result by 2 methods</u>



Kane's Eqns with customary generalized speeds	**44.4 sec**
<u>Recursive Eqns with efficient generalized speeds</u>	**21.0 sec**

Perils of Premature Linearization for Small Motion : Use of Vibration Modes, Can Lead to Loss of Stiffness



Premature linearization occurs if partial velocity in Kane's eqn is derived from linear velocity. Recall Kane's Eqns:

$$0 = \sum_{k=1}^{NP} [\mathbf{F}^k + (-m^k \mathbf{a}^k)] \bullet \frac{\partial \mathbf{v}^k}{\partial u_i} \quad , i = 1, \ldots, n$$

Linearized velocity of P in N for small angle Q1

LVPN> = -OMEGA*L*Q1*A1> + ((L+R)*OMEGA+U1*L)*A2>: **PV>=: L*A2>**

LACC>=(…)*A1>+((L+R)*OMEGA'+U1'*L-OMEGA^2*L*Q1)*A2>

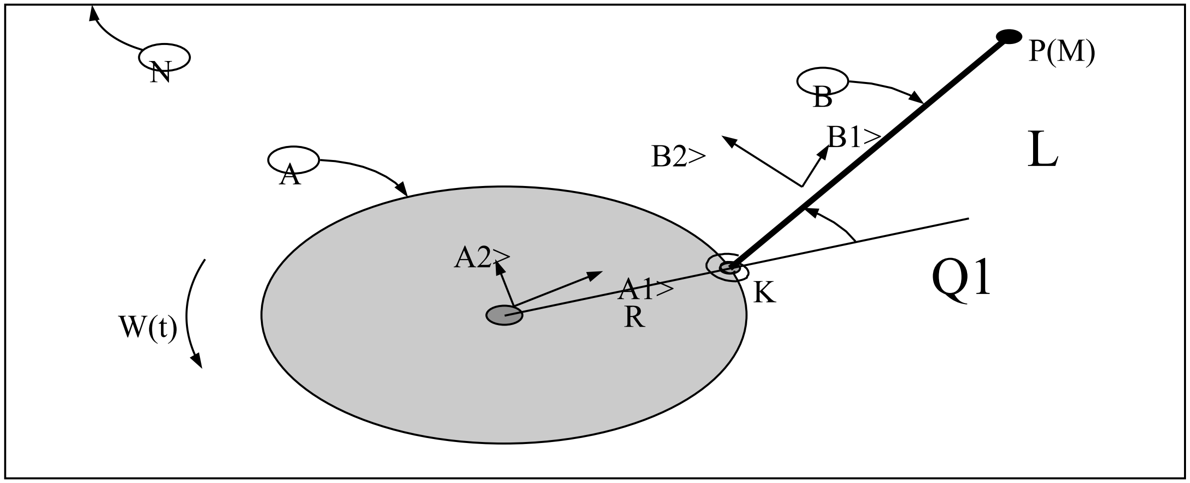
M*LACC> . PV>=-K*Q1 (Kane's Eqn)

U1' = -[K/(M*L^2) -OMEGA^2]*Q1 - (1+R/L)*OMEGA' **wrong eqn !**

Stiffness decreasing with speed, possibly going negative !

Correct Linearization Without Deriving Full Nonlinear Equations

Small vibration of pendulum with large base rotation



Linearize partial velocity from nonlinear velocity expression.
(<u>Beam</u> paper: Kane, Ryan & Banerjee, JGCD, '87; <u>Plate</u> Paper: Banerjee & Kane, JAM,' 89)

Nonlinear expression of velocity of P in N

NLVPN> = OMEGA*R*A2> + (OMEGA+U1)*L*(-sin(Q1)*A1>+cos(Q1)*A2>)

PVN>= L*(-sin(Q1)*A1>+cos(Q1)*A2>);

<u>**Now**</u> **linearize**: LPV>=**L*(A2>-Q1*A1>)** **Premature. linear par. vel** PV>=: **L*A2>**

LVPN> = -OMEGA*L*Q1*A1> + ((L+R)*OMEGA+U1*L)*A2>

ACC>=-A1>*(..+(L+R)*OMEGA^)+((L+R)*OMEGA'+L*(U1'-OMEGA^2*Q1)*A2>

U1' = -[K/(M*L^2) +(R/L)*OMEGA)^2]*Q1 - (1+R/L)*OMEGA' **correct eqn**

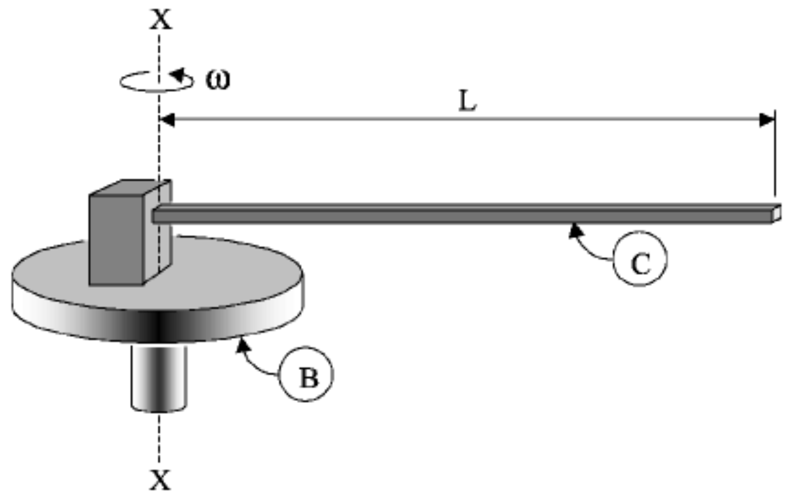
<u>***This approach is**</u> <u>**not feasible for general elastic continua.**</u>

Spin-up Maneuver of Attached Base

Good Test Case for Centrifugal Stiffening of Beams, Plates, and Arbitrary Flexible Structures: Spin-up Maneuver of the Attached Base

$$\omega = \frac{\Omega}{T}\left[t - \frac{T}{2\pi}\sin\frac{2\pi t}{T}\right], \quad t < T$$

$$= \Omega, \qquad t > T$$



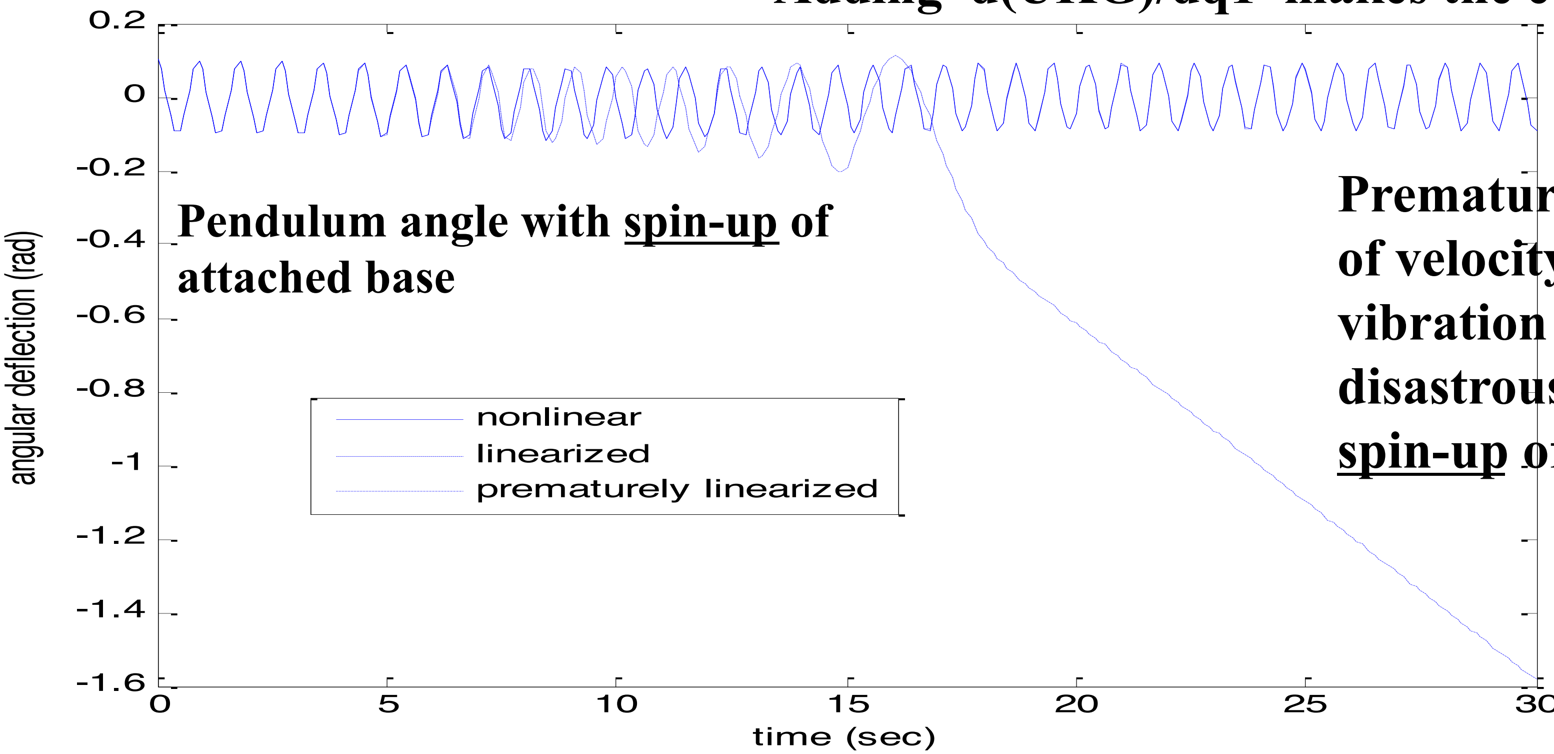
Relevant for Helicopter Rotor Spin-up & Steady Spin

Redeeming Prematurely Linearized Eqns by Adding Geometric Stiffness due to Inertia Loads: Makes Up for Loss of Stiffness

UKG=0.5*M*OMEGA^2*(R+L)/L*[0,q1]*[1;-1;-1,1]*[0;q1]   **Potential for KG of bar* due to inertia load**

U1' = [- K/(M*L^2) + OMEGA^2 - (1+R/L)*OMEGA^2]*q1 - (1+R/L)*OMEGA'

Adding d(UKG)/dq1 makes the equation correct



Pendulum angle with spin-up of attached base

Premature linearization of velocity for small vibration leads to disastrous results for spin-up of base

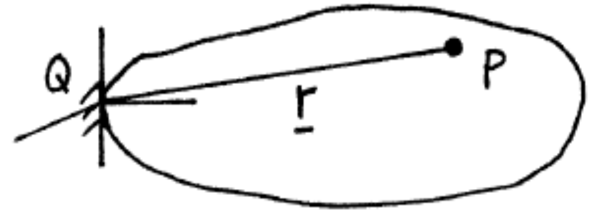
$$UKG = \frac{AE}{2}\int_0^L [\frac{\partial u_x}{\partial x}]_0 (\frac{\partial u_y}{\partial x})^2\, dx = \begin{bmatrix} q_1 & q_2 \end{bmatrix} (\frac{F}{L}) \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix}$$

[Przemieniecki]*

USING VIBRATION MODES IN LARGE OVERALL MOTION IS AN ACT OF PREMATURE LINEARIZATION: A GENERAL THEORY OF MOTION-INDUCED STIFFNESS CORRECTS UNAVOIDABLE ERROR IN EQUATIONS

BANERJEE & DICKENS USES PRECOMPUTED GEOMETRIC STIFFNESS DUE TO BASE ACCELERATION FOR GENERAL CONTINUA

$$a^P = a^Q + \tilde{\alpha}\, r + \tilde{\omega}\,\tilde{\omega}\, r$$



$$\begin{Bmatrix} f_1 \\ f_2 \\ f_3 \end{Bmatrix} = -\,dm \begin{bmatrix} 1 & 0 & 0 & x & 0 & 0 & y & 0 & 0 & z & 0 & 0 \\ 0 & 1 & 0 & 0 & x & 0 & 0 & y & 0 & 0 & z & 0 \\ 0 & 0 & 1 & 0 & 0 & x & 0 & 0 & y & 0 & 0 & z \end{bmatrix} \begin{Bmatrix} a_1 \\ \vdots \\ a_{12} \end{Bmatrix}$$

Compute KG from FEM for inertia load on a point mass at (x,y,z)

$$F = -\sum_{I=1}^{12} a_i\, \Phi^T K_i^g\, \Phi\, \eta$$

Gen. force for motion-induced stiffness

<u>Add geometric stiffness for</u> <u>12 sets of motion-induced loads, for general motion of frame, to overcome loss of stiffness inevitably occuring with use of vibration modes</u>

“Dynamics of an Arbitrary Flexible Body in Large Rotation and Translation” by Banerjee & Dickens, JGCD,’ 90, where acceleration terms, a1,…,a12 are identified. See results on next slide:

BEAM & PLATE SPECIAL THEORIES COMPARED AGAINST GENERAL THEORY WITH MOTION-INDUCED STIFFNESS : case of BASE SPIN-UP

Cantilever beam, Freq 1 = 0.546R/s, Omega = 0.76R/s, T = 1200 s

Beam
tip defln
by two
theories

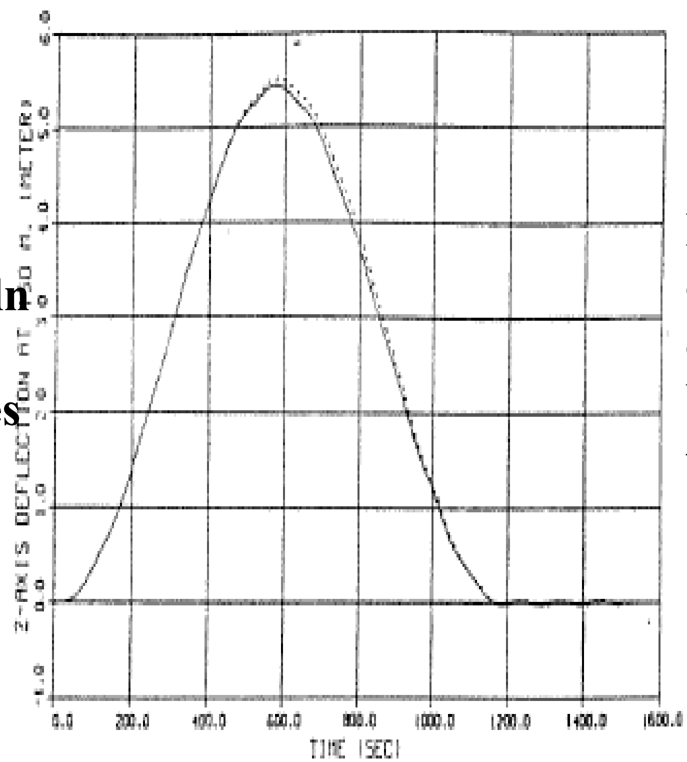


Fig. 6 Cantilever beam tip deflection during spin-up given by the present theory (solid line) and the theory of Ref. 1 for steady-state spin frequency greater than the first vibration mode frequency.

Cantilever plate, Freq 1 = 0.75R/s, Omega 1 = 1.25R/s T = 30 s

Plate
corner
defln
by two
theories

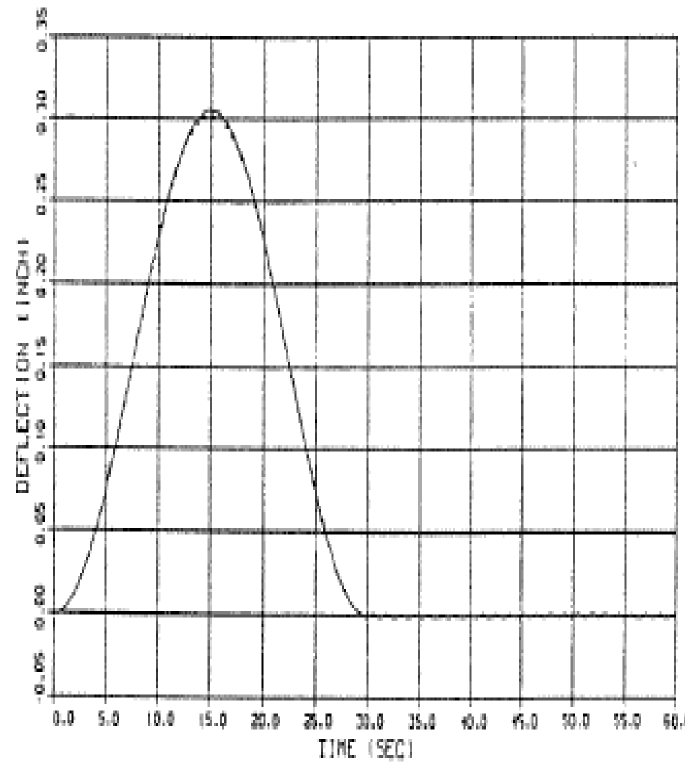


Fig. 7 Cantilever plate corner deflection during spin-up given by the present theory (solid line) and the theory of Ref. 3 for steady-state spin frequency greater than the first vibration mode frequency.

Geo. stiff. theory & special* beam and plate theories agree

***Beam neutral axis stretches [JGCD,' 87]** ***Plate midsurface does not stretch[JAM,' 89]**

Flexible Spinning Paraboloidal Antenna of Dual-Spin Spacecraft

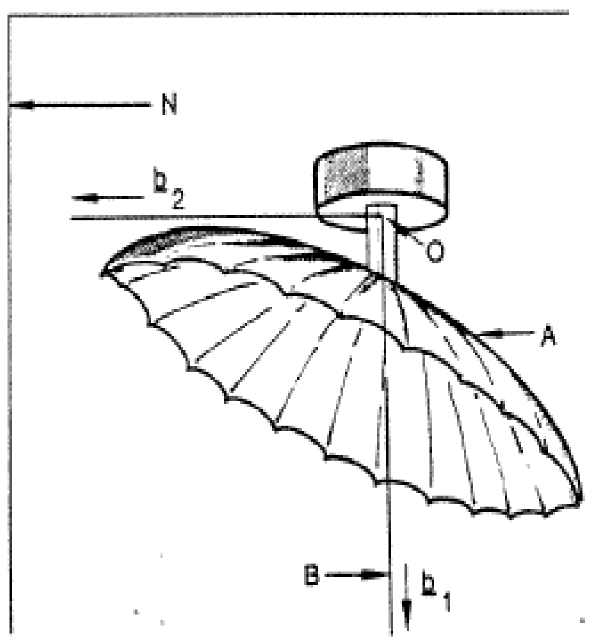


Fig. 1 Dual-spin spacecraft with flexible, spinning offset paraboloidal antenna.

Shell node defln in spin-up by two theories

Paraboloidal antenna, Freq 1 = 0.433 Hz, Omega = 0.5 Hz, T = 30 s

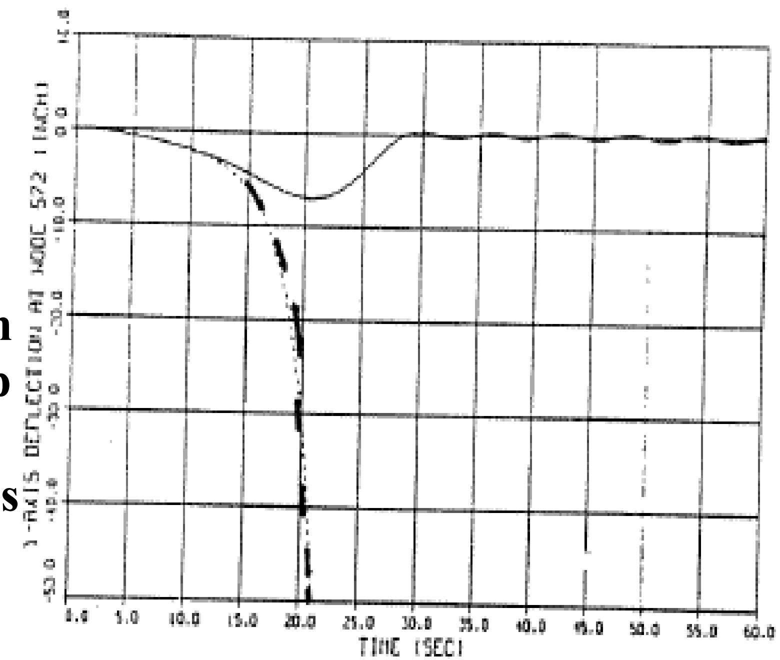
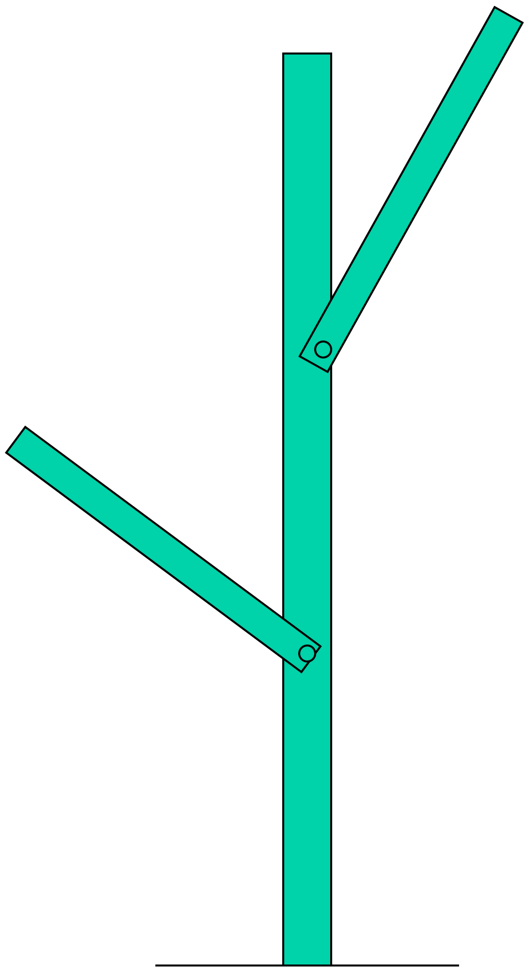


Fig. 2 Elastic displacement along *y* axis at a finite-element node of paraboloidal antenna for spin-up given by present theory (solid line) and a conventional theory (dashed line); steady-state spin frequency is greater than the first mode vibration frequency.

Spin-up of paraboloid shell model: <u>KG-theory works,modal theory fails</u> [Banerjee & Dickens, JGCD,’ 90, pp. 221-227]

Model Reduction for Articulated Flexible Bodies : 3 Beam Example



- **<u>Three hinge-connected elastic beams</u> with actuator and sensor at joints**

$$\left[\quad \mathbf{M} \quad\right]\begin{Bmatrix}\ddot{q}_1\\ \ddot{\theta}_1\\ \ddot{q}_2\\ \ddot{\theta}_2\\ \ddot{q}_3\end{Bmatrix}+\left[\quad \mathbf{K} \quad\right]\begin{Bmatrix}q_1\\ \theta_1\\ q_2\\ \theta_2\\ q_3\end{Bmatrix}=\left[\quad\right]\{u\}$$

We want a select a set of component modes that participate significantly in the system modes

Flexible Body Model Reduction by Singular Value Decomposition of Projected SYSTEM Modes: Three Beam Example

Elastic deformation given by component modes times modal coord

$$\delta_1 = \phi_1 q_1 \,,\ \delta_2 = \phi_2 q_2 \,,\ \delta_3 = \phi_3 q_3$$

Component modal coordinates related to system modal coordinates

$$\begin{Bmatrix} q_1 \\ \boldsymbol{\theta}_1 \\ q_2 \\ \boldsymbol{\theta}_2 \\ q_3 \end{Bmatrix} = \begin{bmatrix} A \\ \\ B \\ \\ C \end{bmatrix} \{\boldsymbol{\eta}\};$$

A, B, and C are projections of system eigenvectors on component modal coordinate subspace

$$[U_1, \Sigma_1, V_1] = svd(A); \quad [U_2, \Sigma_2, V_2] = svd(B); etc.$$

$$\boldsymbol{\delta}_1 = \boldsymbol{\phi}_1 \hat{U}_1 \boldsymbol{\eta}_1 = \hat{\boldsymbol{\phi}}_1 \boldsymbol{\eta}_1; \quad \boldsymbol{\delta}_2 = \boldsymbol{\phi}_2 \hat{U}_2 \boldsymbol{\eta}_2 = \hat{\boldsymbol{\phi}}_2 \boldsymbol{\eta}_2; \quad etc.$$

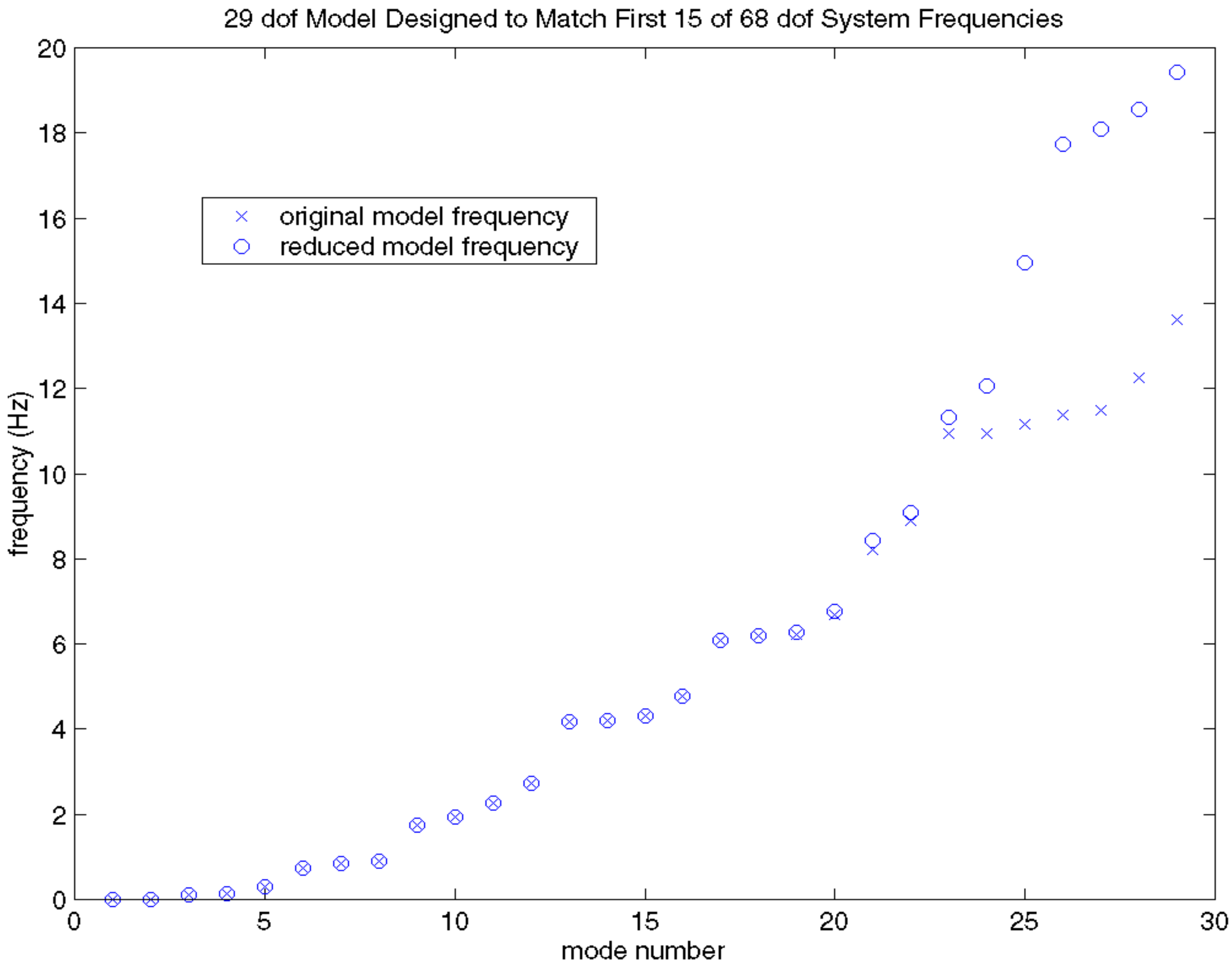
$$q_1 = A\boldsymbol{\eta}$$

$$q_2 = B\boldsymbol{\eta}$$

$$q_3 = C\boldsymbol{\eta}$$

Do SVD of A,B,C and keep as many columns of U as the ranks (SV) of A,B,C to modify comp modes

Results of 29 dof model designed to match first 15 freqs of 68 dof articulated flexible body system for 3 beam model [Lemak]



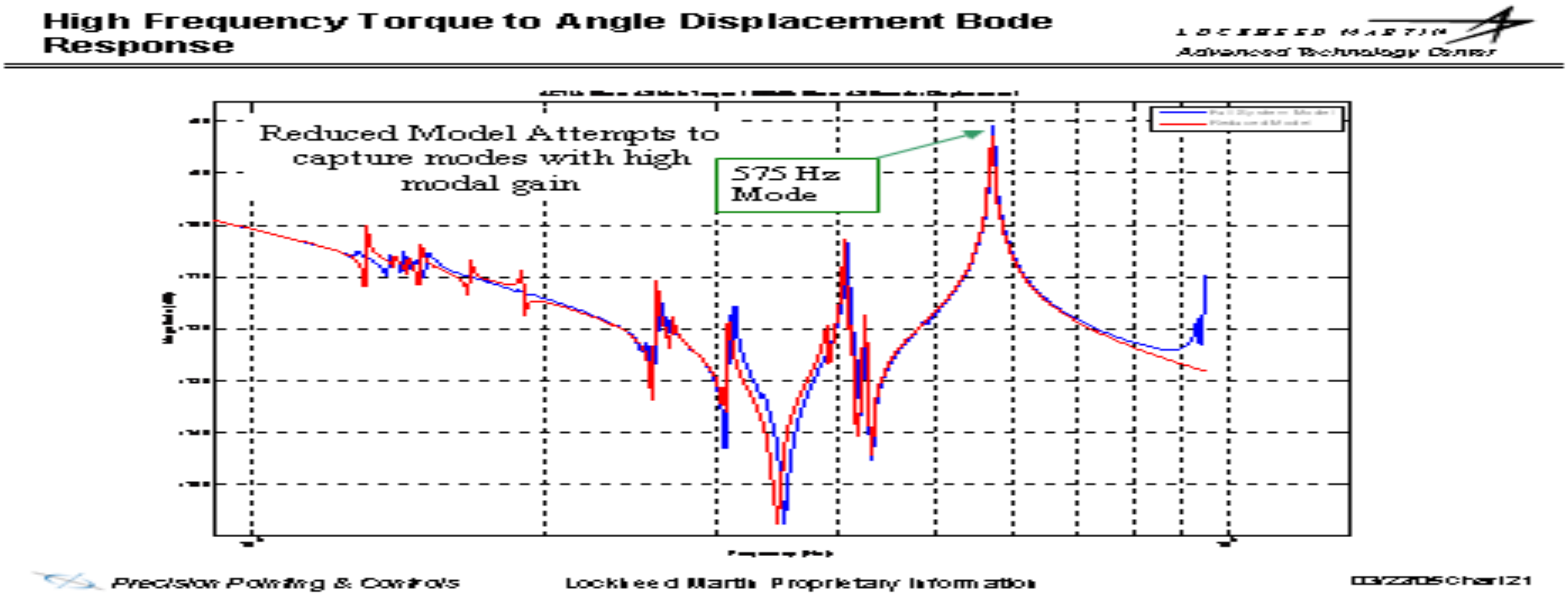
$$\delta_1 = \phi_1 \hat{U}_1 \eta_1$$

$$\delta_2 = \phi_2 \hat{U}_2 \eta_2$$

$$\delta_3 = \phi_3 \hat{U}_3 \eta_3$$

Out of 15 system modes, SVD kept <u>15</u> modes in matrix A of body 1, <u>5</u> modes in B of body 2, and <u>9</u> modes in C of body 3, to get a (15+5+9)dof model

Bode Plot for Reduced & Full Order Model in <u>An Actual Application</u> [Lemak]



Body	SC Bus	Scan. Pivot Tube	Scan. Mirror	Scan. Az Wheel	Scan. El Wheel	Starer Pivot Tube	Starer Mirror	Starer Az Wheel	Starer El Wheel
Full System Modes	*894	30	9	3	3	30	9	3	3
<u>Reduced Model Modes</u>	*31	4	2	3	3	4	2	3	3

Deriving Damping of Components from System Level Damping

Assume System Damping Factor ζ

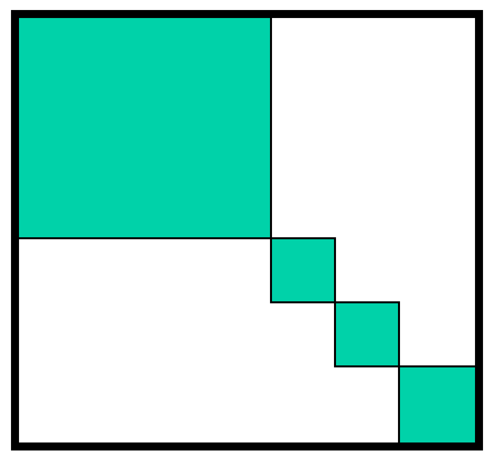
$$C = \Phi^{-t}\left[2\,\varsigma\,\omega\right]\Phi^{-1}$$

$$\Phi^{-1} = (\Phi^{t}\Phi)^{-1}\Phi^{t}$$

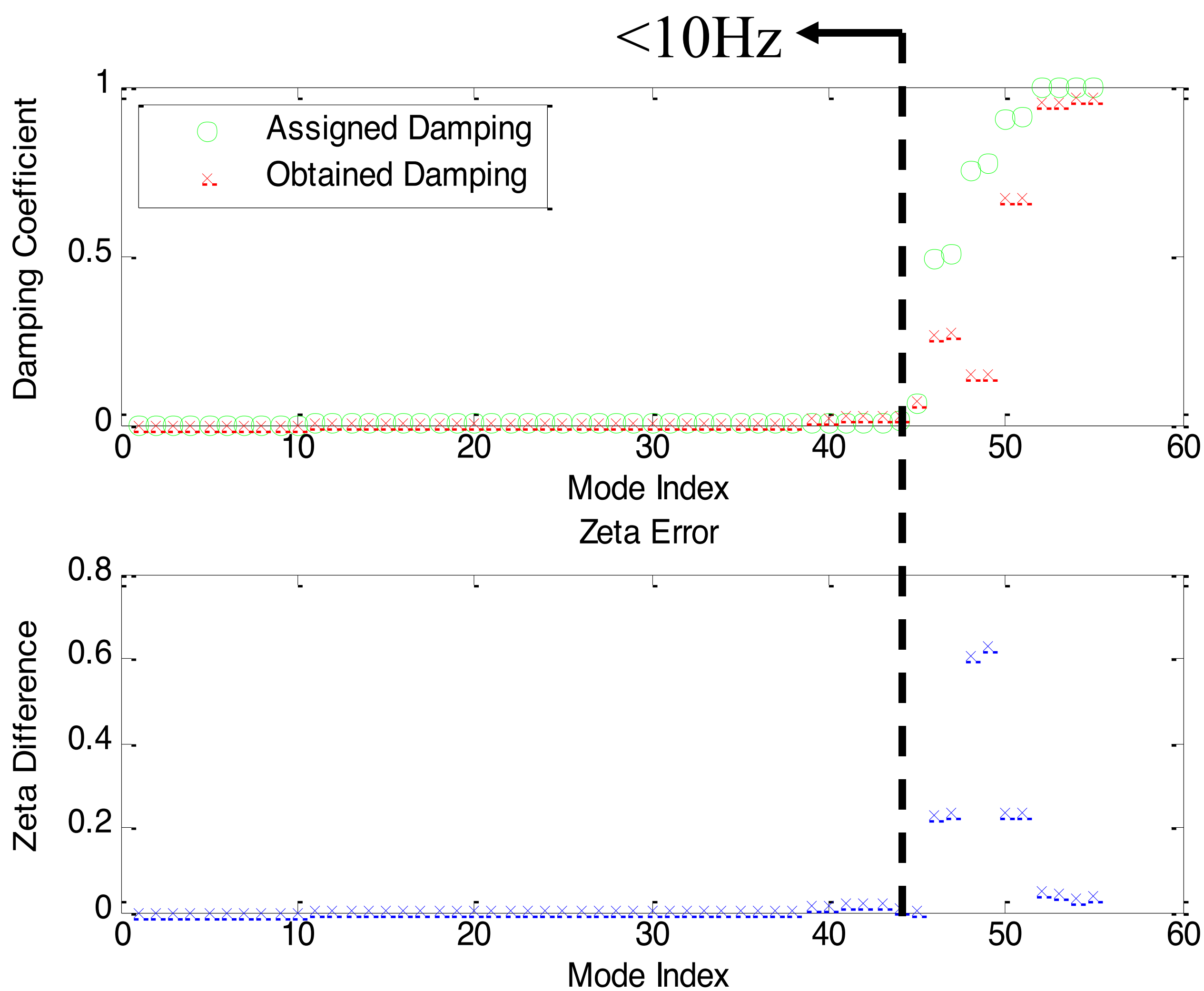
pseudo-inverse
least square err

Derive component damping matrix,
with S_j matrix selecting component j

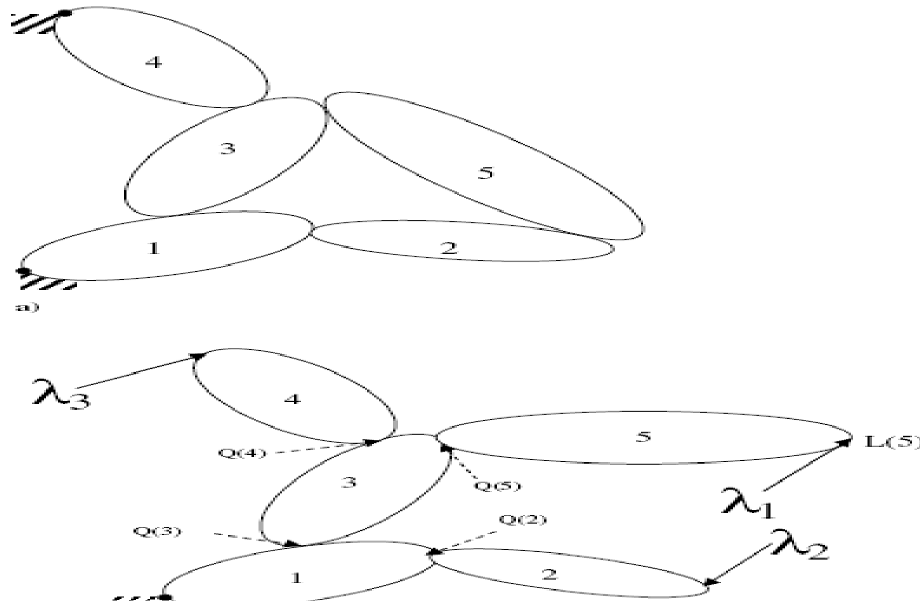
$$c_j = S_j\; C\; S_j^{t}$$



Result of System Damping Re-synthesis with Derived Component Damping in an Application [Lemak]



Recursive Formulation for Constrained Mechanical Systems

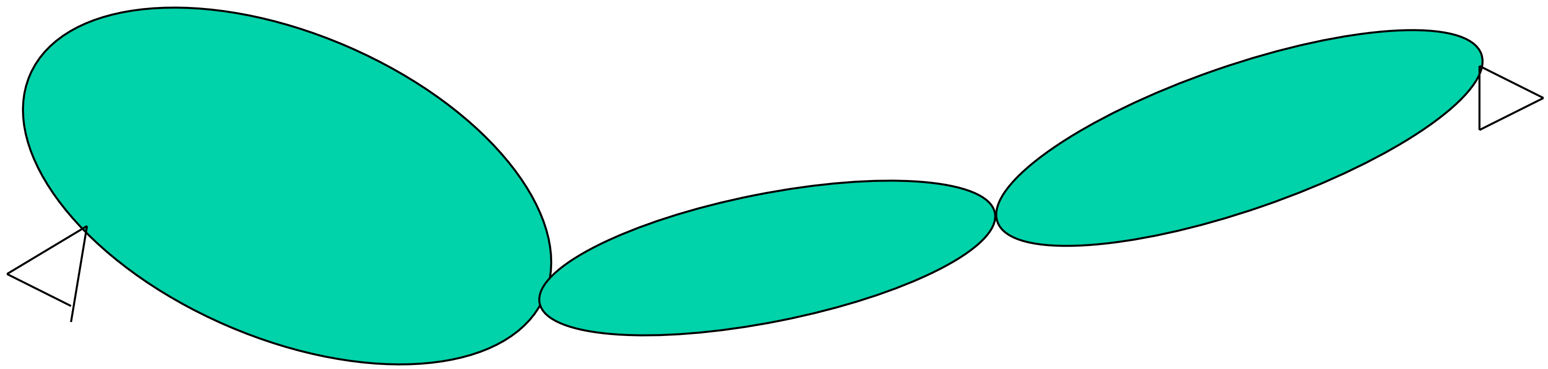


Closed loop system is cut to form open loop tree system: then add constraints.

1. **Cut Loops and Make Forward Pass for Kinematics of Tree System so Formed**
2. **Do Backward Pass Tracking Part of Contributions from Constraint Forces**
3. **Complete Second Forward Pass with Constraint Forces in Dynamical Eqns**
4. **Add Constraint Conditions & Solve Dynamical & Constraint Eqns**

Efficient Generalized Speeds, Recursive Formulation, and Multi-Point Constraints in Flexible Multibody Dynamics-Banerjee & Lemak, JGCD, '07

Constrained Dynamical Equations: Adding Extra DOFs and Constraint Eqns --- Two Formulations



Order-n^3 <u>Extended Kane Eqns</u> **<u>New Order-n Eqns</u>:**

<u>Both methods expose constraint force by cut & use constraint eqns</u>

$$\begin{bmatrix} m_1 & m_2 & m_3 & a^t \\ m_2^t & m_4 & m_5 & b^t \\ m_3^t & m_5^t & m_6 & c^t \\ a & b & c & 0 \end{bmatrix} \begin{Bmatrix} \dot{u}_1 \\ \dot{u}_2 \\ \dot{u}_3 \\ \lambda \end{Bmatrix} = \begin{Bmatrix} f_1 \\ f_2 \\ f_3 \\ d \end{Bmatrix}$$

$$\begin{Bmatrix} \dot{u}_1 \\ \dot{u}_2 \\ \dot{u}_3 \end{Bmatrix} = \begin{Bmatrix} g_1 \\ g_2 \\ g_3 \end{Bmatrix} + \begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix} \{\boldsymbol{\lambda}\}$$

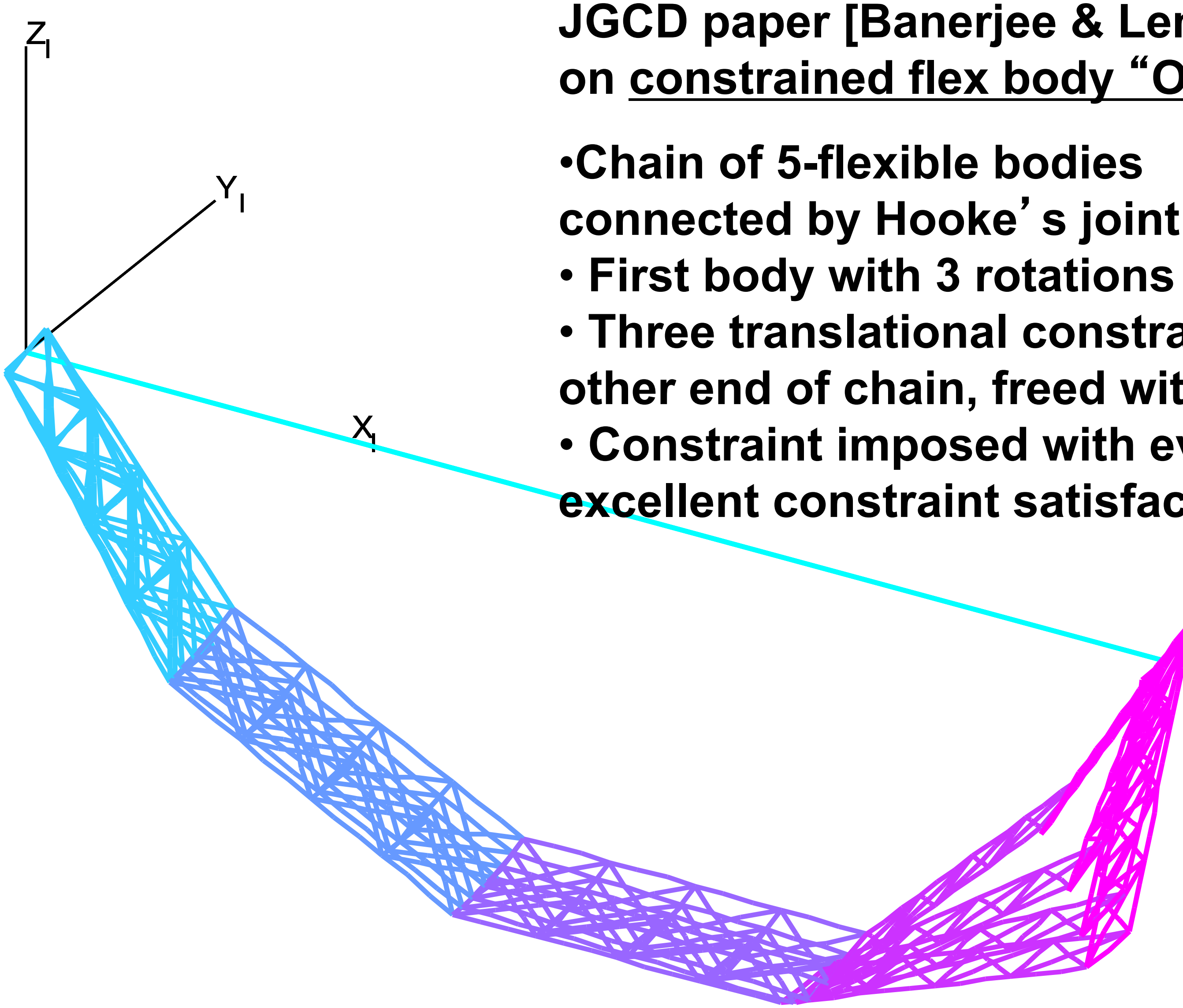
$$a\dot{u}_1 + b\dot{u}_2 + c\dot{u}_3 = d$$

CPU Results for 8 Bodies, 3 LOOPS, Varying # Modes [Lemak]

*Note: Absolute CPU time will go down with use of faster computers

Number of Modes per Body	Number of Generalized Speeds	CPU sec for 10 sec Extended Kane Formulation*	Ratio of Ext. Kane over Efficient Order-n Formulation
0	24	6.28	0.969
4	56	57.57	1.516
8	88	909.29	3.053
12	120	2484.3	4.455

Stroboscopic Plot of a Whirling Chain of Five Articulated Elastic Trusses, 75 dof with 3 constraints [Lemak]



JGCD paper [Banerjee & Lemak,' 07] on <u>constrained flex body "O(n)" eqns</u>

- **Chain of 5-flexible bodies connected by Hooke's joint**
- **First body with 3 rotations**
- **Three translational constraints at other end of chain, freed with force**
- **Constraint imposed with eventual excellent constraint satisfaction**

Kane's Equations for Variable Mass Flexible Body Dynamics [Banerjee, Int. Cong. Th. & App. Mech, '08, Adelaide]

$$\int_0^{m(t)} {}^N\mathbf{v}_i^P \cdot {}^N\mathbf{a}^P \, dm =$$

$$\sum_{k \in L} {}^N\mathbf{v}_i^k \cdot \underbrace{(-\dot{m}_k \, \mathbf{v}_e)}_{\text{Thrust term}} + \int_S {}^N\mathbf{v}_i^P \cdot d\mathbf{f}_{ext}$$

$$- \frac{\partial \pi}{\partial q_i} - \frac{\partial D}{\partial \dot{q}_i} \; ; i = 1,..,6 + n$$

RHS has Thrust, External Forces, & Forces from Potential π due to Structural Stiffness & Geometric Stiffness due to Thrust , & Dissipation Functions *D*.

Form of (6+n) DOF Flexible Rocket Dynamics Eqns

$$\begin{bmatrix} I^{(1)} & -\tilde{I}^{(2)} & I^{(4)} \\ \tilde{I}^{(2)} & I^{(3)} & I^{(5)} \\ I^{(4)^t} & I^{(5)^t} & I^{(7)} \end{bmatrix} \begin{Bmatrix} \dot{V} \\ \dot{\omega} \\ \dot{\sigma} \end{Bmatrix} + \begin{Bmatrix} I^{(1)}\,\tilde{\omega}\,V + \tilde{\omega}\,(\tilde{\omega}\,I^{(2)} + I^{(4)}\,\sigma) \\ I^{(2)}\,\tilde{\omega}\,V + \tilde{\omega}\,I^{(3)}\,\omega + \sum_{j}^{n} I_j^{(8)^t}\,\sigma_j\,\omega \\ I^{(4)^t}\,\tilde{\omega}\,V + W \end{Bmatrix} = \begin{Bmatrix} R_1 \\ R_2 \\ R_3 \end{Bmatrix} + \begin{Bmatrix} U \\ \tilde{r}_t \\ \phi^t \end{Bmatrix} \dot{m} V_e$$

Dynamical Eqns Involve <u>Variable Mass Modal Integrals</u>, $I^{(j)}$

States are frame origin velocity, angular velocity, efficient modal generalized speeds;
Effects of thrust show up in generalized force and load-dependent geometric stiffness of structure, which affects frequencies.

Computing Variable Mass Modal Integrals in Rocket Dynamics

Approximation:
Modes Not Changing Spatially with Time: simplifies time-varying modal integrals via interpolation

$$I_j^{(k)} = \int_0^{m(t)} F^{(k)}\,(\,r_0\,,\boldsymbol{\phi}_j\,)\,dm$$

$$= g(t)\int_0^{m_f} F^{(k)}\,(\,r_0\,,\boldsymbol{\phi}_j\,)\,dm_f$$

$$m(t) = m_f\, g(t)$$

$$g(t) = \left\{ \frac{m_0}{m_f} - \frac{\dot{m}_0\, t}{m_f} + \left(\frac{t}{t_f}\right)^2 \left[3 + 2\,\frac{\dot{m}_0\, t_f}{m_f} - 3\,\frac{m_0}{m_f}\right] - \left(\frac{t}{t_f}\right)^3 \left[2 + \frac{\dot{m}_0\, t_f}{m_f} - 2\,\frac{m_0}{m_f}\right] \right\}$$

Mass at time t modeled by Hermite polynomial in terms of initial mass, mass loss rate, and final mass

Commanded Gimbal Angle, Altitude, Horiz. Disp, Pitch Angle (ICTAM '08)

Command gimbal angle

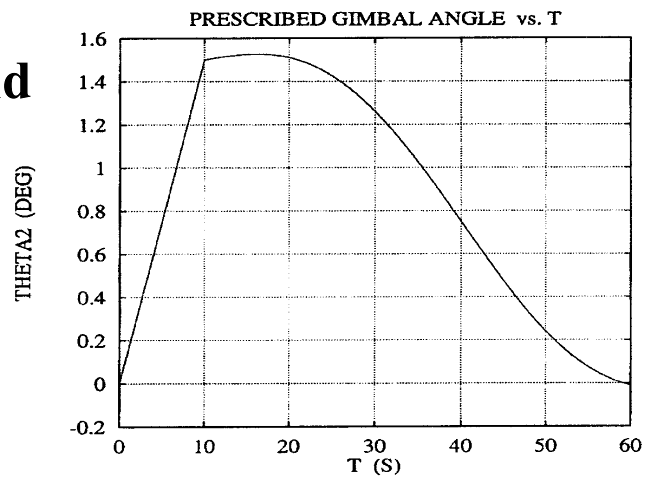


Fig. 2 Prescribed motion time history of the nozzle gimbal angle.

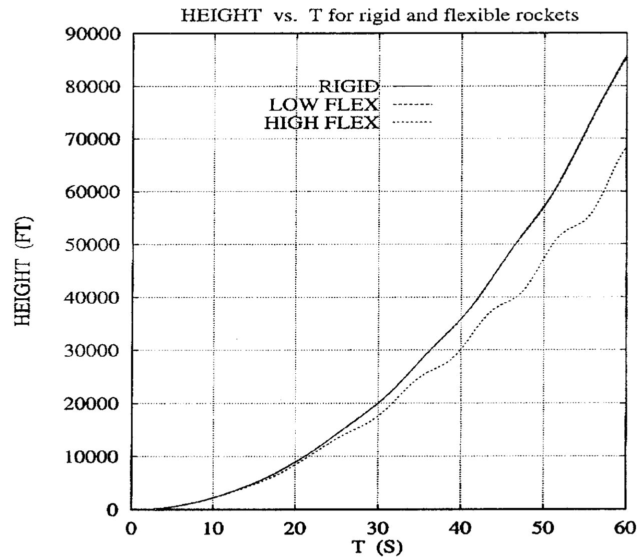


Fig. 3 Height of the rocket from the ground vs time, given by three rocket models.

Rocket vertical disp less for high flex

Rocket horiz displ more for high flex

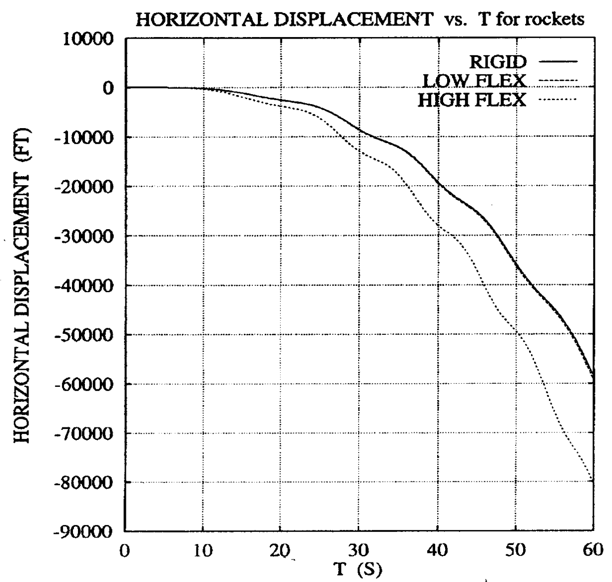


Fig. 4 Horizontal displacement of the rocket vs time, given by three rocket models.

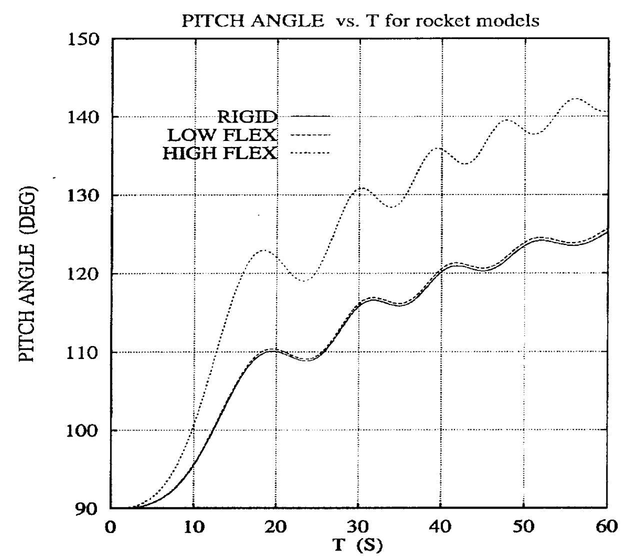


Fig. 5 Pitch angle of the rocket vs time, given by three rocket models.

Rocket pitch more for high flex

Tip Defl.,First Mode Freqs,Torque for Prescribed Motion (ICTAM ‘08)

Tip & mid-point (---) defl of high flex rocket

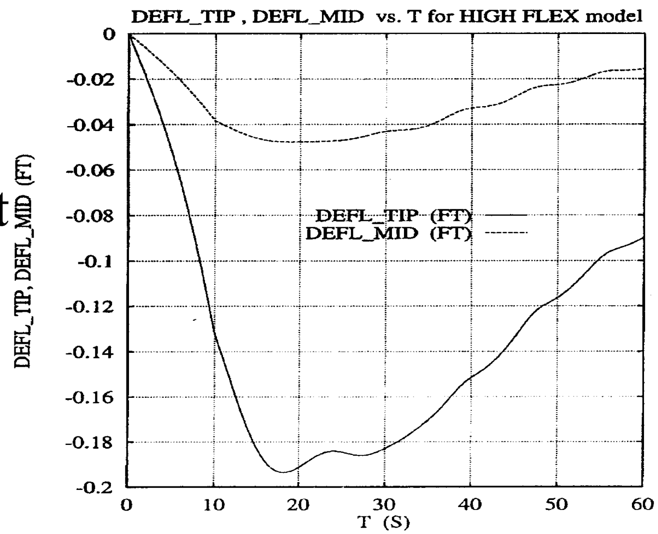


Fig. 6 Tip and midpoint deflection of the high-flexibility rocket mod vs time.

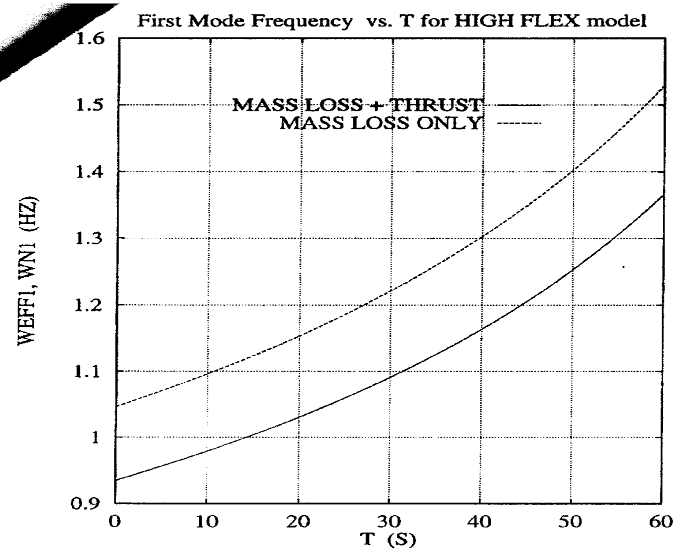


Fig. 7 First mode frquency of high-flexibility rocket model vs time with mass loss only (dotted line) and under the combined action of mass loss nd thrust.

First mode freq for high flex rocket with/wo thrust

First mode freq for low flex rocket w/wo thrust

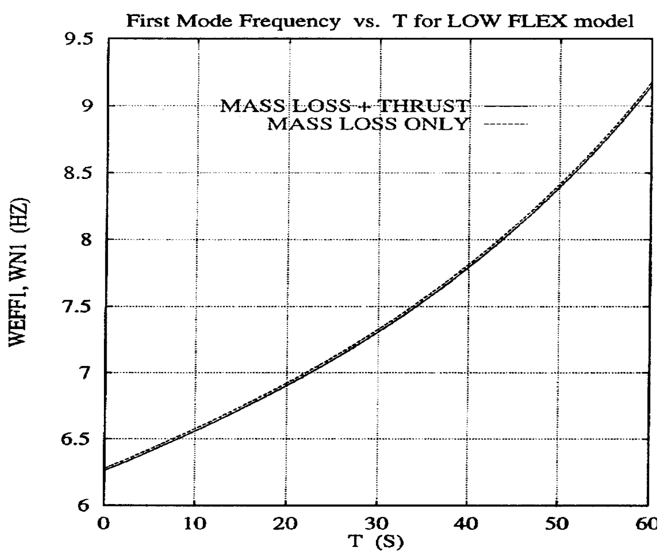


Fig. 8 First mode frquency of low-flexibility rocket model vs time mass loss only (dotted line) and under the combined action of mass and thrust.

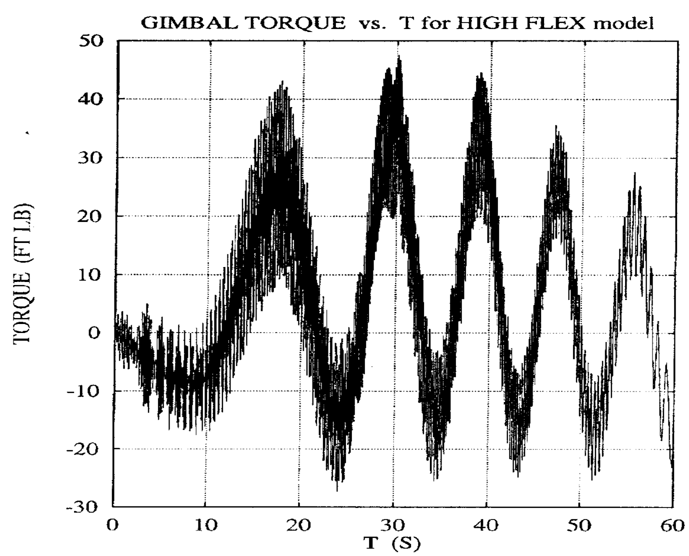


Fig. 9 Gimbal torque in the high-flexibility rocket simulation with specified gimbal motion vs time.

Required gimbal torque for gimbal command angle (high flex case)

CONCLUSION: High-Fidelity, Time-Efficient Modeling of Large Overall Motion of Flexible Multibody Systems

1. Computational efficiency increases with i) Block-diagonal recursive Kane formulation, ii) Choosing generalized speeds to simplify dynamical eqns, iii) Representative modal reduction.
2. Geometric stiffness for 12 inertia loads corrects unavoidable error of premature linearization in using vibration modes.
3. Large deflection treated by O(n) formulation with spring-connected rigid multibody models, producing as high-fidelity results as nonlinear FEM, & being more time-efficient.
4. Variable-n O(n) method models deployment / retrieval of beams and cables with fidelity and efficiency.
5. For systems with closed structural loops the recursive method, employed by cutting loops & solving constraint forces, is efficient.
6. Kane's eqns for variable mass flexible bodies, modal integrals via Hermite interpolation, with freqs modified by thrust, are given.

UPCOMING BOOK: FLEXIBLE MULTIBODY DYNAMICS, BANERJEE: WILEY PUBLICATIONS, N.Y., 2016